

Stock-Flow Analysis in Consumption and Household Production Theory: A Review and Synthesis

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Hicks observed some time ago that "it is only if the two match: if the acquisition of new consumption goods just matches the using up of old ones that we can say consumption equals spending"¹. But having thus recognized consumer inventory holdings, Hicks immediately disposed of the problem again as a theoretical nicety. This view of the problem must have been widely shared by economists since inventory holdings are still absent from conventional consumption models.

In the last decade, there have been several innovations in demand theory which have a bearing on the development of a consumer inventory model. Chronologically enumerated they are found in the works of Clower², Becker³ and Lancaster⁴ and in subsequent responses by others⁵. Since the approaches are related in the way they analyze household consumption, it will be useful to try and synthesize the various contributions.

Before reviewing these major models, however, a recent dissertation by S. Sharir on "Consumer Buying Behaviour and the Structure of Retail Industries,"⁶ is of interest and should be discussed. It fully recognizes the consumer inventory problem within the wider context of an allocation of time analysis of the marketing activities of an economy. Goods have to be transferred from the point of production to the point of consumption, a service which Sharir terms "Commercial Distribution". The service is in part provided by the retail industry and in part by the household. Its total value is measured by the sum of the time which households

¹ J. R. Hicks, *Value and Capital*, at the Clarendon Press, Oxford, 1939, p. 176.

² See for example: D. W. Bushaw and R. W. Clower, *Introduction to Mathematical Economics*, R. D. Irwin, Homewood, Ill., 1957; R. W. Clower, "Stock Flow", *Encyclopedia of the Social Sciences*, Collier MacMillan, New York, 1966; R. W. Clower, "On the Technology of Exchange", unpublished paper, 1969; R. W. Clower, "Is there an Optimal Money Supply?", *Journal of Finance*, Vol. XXV (June 1970).

³ G. S. Becker, "A Theory of the Allocation of Time", *Economic Journal*, LXXV (Sept. 1965).

⁴ Lancaster, "New Approach".

⁵ See for example: S. Sharir, *Consumer Buying Behaviour and the Structure of Retail Industries*, unpublished Ph. D. dissertation, Columbia University, 1970; R. Gronau, *The Value of Time in Passenger Transportation: The Demand for Air Travel*, *National Bureau of Economic Research*, occasional paper 109, New York, 1970; J. Hadar, "A Note on Stock-Flow Models of Consumer Behaviour", *Quarterly Journal of Economics*, LXXIV (1965); R. G. Lipsey and G. Rosenbluth, "A Contribution to the New Theory of Demand: A Rehabilitation of the Giffen Good", *Canadian Journal of Economics*, IV (May 1971); J. D. Owen, *The Price of Leisure, An Economic Analysis of the Demand for Leisure Time*, McGill - Queens University Press, 1970.

⁶ Sharir, op. cit.

use to provide the service for themselves and by the amount of time households save because retailers provide the rest of the service.

Households periodically sacrifice time on shopping trips to collect commodities and then hold these goods in inventories. The model treats shopping time, and the resources used up in holding goods, as substitutable inputs and thus recognizes a basic property of household inventory behaviour. However, by analyzing the problems involved in allocating the total time required for "Commercial Distribution" Sharir is confronted with many issues additional to those arising at the household level. His dissertation thus studies a different dimension of the problem.

Clower's Stock-Flow Analysis

In 1959, Clower, in cooperation with Bushaw, developed a stock-flow theory of consumption⁷ and thereby provided the first general treatment of stocks of commodities held by consumers. The theory asserts that the holding of any commodity facilitates consumption and must therefore yield utility to the consumer. Because utility is derived from consuming as well as holding the commodity, consumer behaviour must be analyzed in terms of his utility received from holding and consuming commodities.

The theory postulates the existence of an optimum inventory stock for each commodity consumed. Therefore, spending is for inventory accumulation as well as for consumption itself.

This theory of consumption lends rigor to demand theory by using the stock-flow analysis as an explanation of consumer spending behaviour. Aside from its theoretical appeal, the stock-flow model also suggests a wealth of realistic approaches to conventional theories as well as to economic phenomena not yet explained.

In a more recent paper "On the Technology of exchange,"⁸ and in a refined follow-up version "Is there an Optimal Money Supply,"⁹ Clower specifies his consumption model in a manner very similar to Baumol's cash inventory model¹⁰. Households want to hold commodity inventories because these holdings enable households to earn income in terms of the leisure gains available from fewer shopping trips. These income benefits will have to be compared to the cost of holding inventories consisting of interest, storage and deterioration costs. Optimum inventory holdings are attained when the marginal income gain is equal to the marginal cost of holding the goods.

⁷ Bushaw and Clower, op.cit.

⁸ Clower, "On Technology".

⁹ Clower, "Is there Optimal Supply?".

¹⁰ W.J. Baumol, "The Transaction Demand for Cash: An Inventory Theoretic Approach", *Quarterly Journal of Economics*, LXI (November 1952).

In treating leisure time explicitly as a scarce resource, Clower is in line with other recent attempts to introduce the cost of time systematically into consumption analysis¹¹. While these papers stress the opportunity cost of time to the exclusion of the other opportunity costs of consumption, Clower's, Sharir's, and Reinhardt's¹² model recognize these costs in their entirety. Just as the procurement of market goods involves costs because it requires the shopper's time, the holding of these goods for consumption will incur a series of holding costs that must also be considered. With this symmetrical treatment of holding costs the stock-flow model contributes to the consistency of the debate on opportunity costs.

Clower's model is deficient on a different score, however. The introduction of an explicit time constraint raises a number of analytical questions which the classical work-leisure analysis is silent on, and which Clower does not seem to realize either.

Becker's Allocation of Time Analysis

In the traditional theory, the consumer finds an optimal division of his total time between leisure and working time, and then the value of leisure is measured in terms of the earnings foregone by not working.

As Becker points out¹³, the approach represents a simplification of a complex decision process that is useful for some problems, but too crude for others. The leisure activity to which the model relates is a case in point.

Far from being a state of suspended animation, leisure time can be spent very productively by earning income in kind. For example, an office worker who exerts himself strenuously at home as handy man, may attain a level of income whose money value exceeds that of his less practical colleague. The latter may earn the same money income from work but prefers to stay idle at home. Clearly, the market opportunity costs of their leisure times must be different at the margin. According to traditional theory, however, the value of time for the two office workers would have to be the same because it would equal their identical wage rates.

It is not even meaningful to talk about a "leisure time", lumping together in it a heterogeneous collection of activities which individuals perform off the job. Only a small, though varying, proportion of non-working time will be used for what is commonly meant by "leisure" or for non-productive consumption, as Becker prefers to call it. During most of the "leisure" time, the household engages in

¹¹ See for example: J. G. Stigler, "The Economics of Information", *Journal of Political Economy*, LXIX (June 1961); J. Mincer, "Market Prices, Opportunity Costs and Income Effects", in *Measurements in Economics*, ed. Christ et al.; Stanford University Press, Stanford, 1963.

¹² P. G. Reinhardt, "A Theory of Household Grocery Inventory Holdings", *Kyklos*, XXVI, No. 3, pp. 497-511.

¹³ Becker, op. cit.

activities that differ greatly in their earnings productiveness. The household, therefore, faces the question of how to allocate time among a multitude of alternative uses to maximize welfare.

Becker solves the problem by treating the household as a production as well as a consumption unit. The household combines market goods X_{ji} and time inputs T_{ji} , $i = 1, \dots, m_j$, of varying productive capacities, to produce commodity Z_j . This commodity then enters the household's utility function as arguments, e.g. $U = U(Z_1, \dots, Z_j)$, where the Z_j 's are consumption activities produced by j production functions of the form $Z_j = Z_j(X_{j1}, X_{j2}, \dots, X_{jm_j}, T_{j1}, T_{j2}, \dots, T_{jm_j})$.

In the short run it is reasonable to argue that the amount of work supplied by the household, T_w , is given. With an exogenously determined amount of time available for consumption, T_c , household production is now subject to two constraints, the total time available for non-work activities

$$T_c = \sum_1^m T_i$$

and the budget constraint I , limiting the total expenditure on market inputs

$$\sum_1^m P_i X_i = w T_w + V \equiv I$$

where w is the market wage rate and V the amount of non-work money income, assumed unrelated to T_c .

The condition for maximum welfare to the household now requires that utility is maximized and that the optimizing set of activities is produced at maximum output combinations of all the X and T inputs.

This condition is derived formally by using the Lagrangian method. According to it a function W is formed

$$W = U(Z_1, \dots, Z_j) + \lambda (I - \sum P_i X_i) + \mu (T_c - \sum T_i)$$

which is maximized by setting its partial derivatives equal to zero:

$$\frac{\partial W}{\partial X_i} = \frac{\partial U}{\partial Z_i} \frac{\partial Z_i}{\partial X_i} - \lambda P_i = 0$$

$$\frac{\partial W}{\partial T_i} = \frac{\partial U}{\partial Z_i} \frac{\partial Z_i}{\partial T_i} - \mu = 0$$

and

$$\frac{\partial W}{\partial \lambda} = \frac{\partial W}{\partial \mu} = 0.$$

The equilibrium condition is therefore

$$\frac{\frac{\partial Z_i}{\partial T_i}}{K} = \frac{\frac{\partial Z_i}{\partial X_i}}{P_i}$$

where K is the imputed price of time which is the ratio of the marginal utilities of time and income, i.e. $\frac{\mu}{\lambda}$.

Lancaster's New Approach to Demand Theory ¹⁴

The analytical relevance of Clower's Stock-Flow and Becker's Allocation of Time Analyses now are greatly enhanced by Lancaster's Household Production Model. It provides a generalized theoretical setting for analyzing household activities, including the particular problems singled out by Clower and Becker.

The approach was prompted by Lancaster's dissatisfaction with conventional consumption theory whose axioms he considers too weak to support any kind of substantive propositions. He sets out by distinguishing a commodity from its consumption characteristics. Commodities are considered agents for the production of consumption characteristics with the latter measurable on a cardinal scale. These characteristics enter the utility function as arguments and it is these measurable characteristics which give rise to utility. The market demand for a commodity is now a derived demand and as such will depend very closely on the other derived demands for commodities producing the same given characteristic. By the same token, this market demand will be largely independent of the goods which produce other unrelated characteristics. For this reason, Lancaster's demand functions contain more definite information than those derived from conventional ordinal utility functions. The reason is that in the latter type of function every good is a potential substitute of another.

Aside from its ability to accommodate the stock-flow mechanism of consumption, and to cope with time as a factor input, Lancaster's approach has another advantage for our purposes that will become apparent shortly. Consumption rates for individual goods, relative to which the inventory optima are determined, are very unstable because many goods are continually substituted for each other. Determining inventory holdings necessary to maintain a given consumption characteristic, may thus be a great deal more useful than finding the holdings for a much larger number of commodities which are consumed very irregularly.

The Relation of Stock-Flow and Household Production Models When Explaining Household Inventory Behaviour

The introduction to consumer analysis of a household production function has the consequence that consumption activities acquire technological properties. By

¹⁴ Lancaster, "New Approach".

viewing the maintenance of an inventory as another service produced and consumed by the household, these production relations can now be specified to describe the stock-flow mechanism of consumption.

Using, as an example, Baumol's simple inventory model, the level of the inventory service can be measured in terms of the number of units q consumed of a good annually which the inventory helps to maintain. Referring to panel (a) in *Figure 1*, total annual procurment costs in dollars are represented by the rectangular hyperbola

$$PC = b \frac{q_0}{Q}$$

as for a given q_0 and a variable purchase size of Q units the annual shopping frequency is $\frac{q_0}{Q}$, letting b represent the total opportunity cost incurred per trip. Holding costs HC in dollars are assumed to increase linearly with the amount of average holdings $\frac{Q}{2}$, so that

$$HC = IP \frac{Q}{2}$$

where I is the total rate at which holding costs accrue annually, and P the unit price of the good.

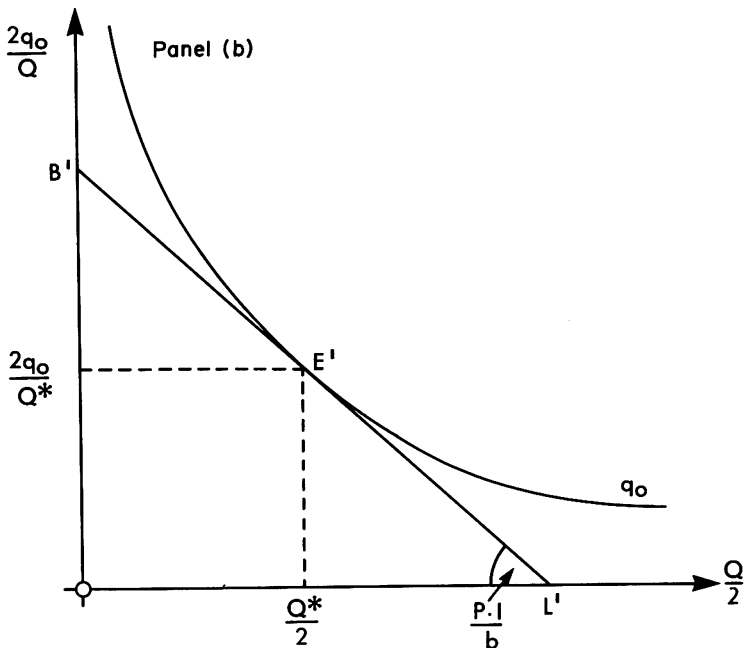
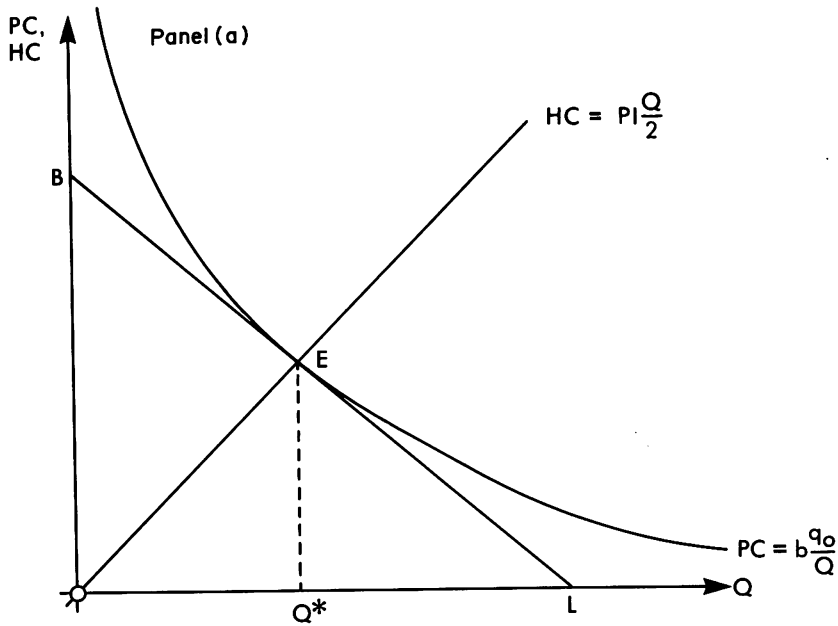
With this specification, inventory equilibrium is achieved where the curves $PI \frac{Q}{2}$ and $b \frac{q_0}{Q}$ intersect. At the point of intersection, the absolute values of the slopes of the two curves are equal. Geometrically this follows because $b \frac{q_0}{Q}$ is a rectangular hyperbola and its slope is always equal to the negative of the slope of any ray from the origin intersecting the hyperbola. The slope of the tangent BL to curve PC at point E has, of course, also the same absolute value as the slope of either cost function. All this is just a diagrammatic interpretation of the condition for total costs to have a minimum, obtained by differentiating the sum $b \frac{q_0}{Q} + PI \frac{Q}{2}$, namely of the condition

$$\frac{PI}{2} = b \frac{q_0}{Q^2}$$

But since we can transcribe this condition as

$$\frac{P \cdot I}{b} = \frac{q_0}{\frac{Q^2}{2}}$$

Figure 1.
The relation of stock-flow and household production models
in explaining household inventory behaviour



we can interpret $\frac{Q^*}{2}$ and $\frac{q}{Q^*}$ as an optimal input combination on a household production isoquant with slope $\frac{q_0}{Q^2}$ for the cost ratio $\frac{P \cdot I}{b}$, as indicated in panel (b) of Figure 1. We merely have to view $\frac{q_0}{Q}$, the annual number of shopping trips, and $\frac{Q}{2}$ the average number of units of inventory of the good as substitutable inputs of the household production function.

It is interesting to note that now cash transactions behaviour is explained by household behaviour in general because Baumol's¹⁵ and Tobin's¹⁶ models can also be conceived as particular specifications of the household production function.

An additional conclusion that may be drawn from this link to well established inventory theory suggests that, at least the inventory aspect of, household production cannot be described by linear homogeneous production functions as Muth¹⁷, for example, has done with household production in general.

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¹⁵ Baumol, op. cit.

¹⁶ J. Tobin, "The Interest Elasticity of the Transactions Demand for Cash", *Review of Economics and Statistics*, XXXVIII (1956).

¹⁷ R. F. Muth, "Household Production and Consumer Demand Functions", *Econometrica*, XXXIV (1966).

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Zusammenfassung

Stock-Flow-Analyse in der Nachfrage- und Produktionstheorie der Haushalte : Überblick und Synthese

Die Abhandlung gibt einen Überblick über neue Beiträge zur Nachfragetheorie, die sich auf das Inventurproblem des Haushalts beziehen. Während Stock-Flow-Analytiker, wie Clower, sich direkt mit dem Sachverhalt befassen, können die Theorien von Becker und Lancaster so angepasst werden, dass sie die Lagerhaltung von Verbrauchern mit einbeziehen.

Indem ein – nach Baumol spezifizierter – Lagerumschlagsmechanismus des Verbrauchs eingesetzt wird, kann in der Abhandlung bewiesen werden, dass dieser Mechanismus mit einem Haushaltsproduktionsmodell äquivalent ist, sofern dieses sich einer besonderen Produktionsfunktion bedient.

Diese vorhandene Beziehung lässt darauf schliessen, dass zwei offenbar unterschiedliche Literaturzweige sich gegenseitig in ihren Verbrauchertheorien bekräftigen.

Résumé

Stock-Flow Analysis dans la théorie de la consommation domestique et de la production ménagère : aperçu et synthèse

Cette étude présente une revue des récentes contributions relatives à la théorie de la demande sous l'aspect du problème de l'inventaire de ménage. Alors que les analystes du stock-flow – Clower notamment – s'attaquent pour leur part directement à la question, les théories de Becker et de Lancaster peuvent toutes deux être adaptées pour comprendre également les provisions gardées en stock par les consommateurs.

Utilisant un type de spécification de Baumol pour le mécanisme du stock-flow dans la consommation, cette étude montre qu'il équivaut à un modèle de production à l'intention des ménages, dans la mesure où il fait usage d'une fonction de production particulière.

Le fait de l'existence d'une telle relation implique que deux tendances littéraires – apparemment séparées l'une de l'autre – se soutiennent mutuellement dans leur approche relative aux problèmes du consommateur.

Summary

The paper presents a review of recent contributions to demand theory that have a bearing on the inventory problem of the household. Whereas stock-flow analysts, notably Clower, address themselves directly to the issue, both Becker's and Lancaster's theories can be adapted to accommodate stock holdings by consumers.

Using a Baumol-type specification of the stock-flow mechanism of consumption, the paper shows that it is equivalent to a household production model when it uses a particular production function.

The existence of this relation implies that two, apparently separate, strands of literature reinforce each other in their treatment of the consumer.