International Migration under Incomplete Information

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Introduction

International migration always has been of utmost importance for the afflicted countries. For example one can consider the immigration waves going from Europe to the United States during the past 300 years. These immigration waves often are regarded as the starting point for the economic development in the United States. But they also created big economic problems in the emigration countries as Italy, Ireland or Greece. To give an example of the international migration in recent time we can mention the so-called "brain drain" or the "guest worker" migration.

The problem of international migration has attracted a large number of economists and social scientists. It is interesting to see that most of them were working empirically.

There is a real lack of theoretical studies in the field of international migration. To be more precise there is a lack of microeconomic migration models giving a precise decision theoretical foundation for the migration decision. Naturally the results of such models would be very interesting for politicians who want to deter or to stimulate migration according to the goals of economic policy.

In the present paper we want to establish a microeconomic decision model for international migration by which it is especially possible to deal adequately with the problem of *incomplete information* of the decision maker. Obviously incomplete information of the migrant about the pecuniary or non-pecuniary benefits in a foreign country is an important aspect of each migration decision. This point also has been emphasized by *M. Greenwood* in his first survey article on migration (1975) where he calls information an important determinant of migration which is lacking of precise theoretical foundation.

In recent years there have been published only few articles on international migration under incomplete information (e.g. *David* (1974), *Stark* (1984), *Kwok/Leland* (1982), *Mayer* (1985), *McCall/McCall* (1984)). Our model is a substantial generalization of *McCall*'s paper who first applied sequential decision modelling (especially Gittins-index theory) to the migration decision.

In detail *McCall* presupposes for each country a stochastic process of net returns on migration characterized by a (stationary) Markovian structure. Naturally this might be a serious restriction for some applications. Beside the usual regularity assumptions (for Stochastic Processes) we do not need in our model any particular assumption concerning the probabilistic dependence of net return streams in the potential immigration countries.

Kwok and *Leland* focus on incomplete information of potential employers about the job productivity of remigrating students having acquired their academic degree in a foreign country. They analyze existence and properties of a sorting equilibrium in the brain drain context. In contrast to *Kwok/Leland*'s approach we focus on modelling incomplete information of an employee about the pecuniary and non-pecuniary characteristics in a foreign country. Secondly, we do not intend in the present paper to bring the two sides of the market (employee and employer) together. Much work has to be done to integrate our sequential decision making approach into an equilibrium model.

Most similar to our model and to *McCall*'s model is the work of *G. Mayer*. He considers a migrant solving a sequential job search problem in each country. Furthermore the migrant is supposed to have an a priori idea of the wage distribution in each country which is adapted (according to Bayes rule) during job search. The main problem with this approach seems to be the following. Remigration in later periods is not taken into account now. The decision maker solves separate problems how to search optimally in one country but he does not fix the optimal time point of remigration. In our opinion this problem has to be treated in a multi-armed bandit framework where the Gittins-index method is an appropriate solution method.

To give a better motivation for our model we will discuss at first (section 1) the so-called "Human Capital model" of migration on which most of the microeconomic models that have been published so far are based. In some sense our model can be viewed as an extension of the Human Capital approach. In section 2 of our paper we will make this idea precise by introducing a sequential decision model for international migration under incomplete information. After establishing the formal model we will discuss Gittins-index theory as a solution method for the migration problem. This method gives an interesting interpretation of the optimal migration strategy. Additionally it will make the comparative statics of the problem considerably easier. This point will be discussed in the final section of our paper where it is demonstrated by a simple example how the optimal migration policy can be influenced by exogenous parameter variation.

I. The Human Capital Model

The Human Capital model of migration is a decision theoretic approach to migration decisions[1] which has been developed at first by *L. Sjaastad* (1962). It has been tested empirically in the seventies (e.g. *Fields* (1979), *Bowles* (1970)). Recent work on this model has been done by *Cebula* (1979).

The basic idea of this approach is rather simple: migration decisions are regarded as specific investment decisions (= investment in Human Capital). Consequently *Sjaastad* proposed to utilize the traditional investment calculus to derive migration decisions. To be more precise a migrant at first is supposed to know return $R_j(t)$ and costs $C_j(t)$ of migration into a specific country j in each future period $t = 1, 2, \ldots$. Here $R_j(.)$ is interpreted to include pecuniary benefits (e.g. real wage) and non-pecuniary benefits (e.g. quality of life, cultural environment) as well. The same is valid for $C_j(.)$ which should be interpreted to include beside transportation costs also opportunity costs of search in the foreign country or so-called "psychic costs" of migration.

Now let us suppose that a migrant has to choose to go into one of N several countries (indexed by $j = 1, \ldots, N$) then he calculates the capital value K_j for each country by[2]

$$K_j := \sum_{t=1}^{\infty} \beta^{t-1} (R_j(t) - C_j(t))$$

where β denotes the (subjective) discount rate of the migrant. Then the migration decision according to the Human Capital approach can be described by the following

Rule I: Suppose there are N countries, each characterized by a time path of returns and costs of migration $\{R_j(t), C_j(t)\}_{t=1}^{\infty}$, then choose country $j^*$where

$$K_{j^*} = \max_{j \in \{1, \ldots, N\}}$$

The Human Capital approach proved to be superior[3] to the "wage differential" hypothesis, dominating till the sixties, which stated that migration decisions mainly depend on actual real wage differences between the countries. Compared with this, in *Sjaastad*'s Human Capital model migration decisions depend on

[1] The model has been developed for *internal* migration decisions but it can easily be extended to *international* migration.

[2] For the sake of simplicity we assume that the decision maker has an infinite planning horizon.

[3] This has been tested empirically many times (e.g. *Bowles* (1970), *Fields* (1979)).

actual *and* on future real wage differences (and migration costs) as well. Furthermore, beside future real wages (= pecuniary returns) also non-pecuniary benefits in the foreign countries are considered by the migrant.

Unfortunately, there is a major problem with *Sjaastad*'s Human Capital approach. As the decision making migrant has to take into account all *future* net returns on migration he has to deal with estimated variables exhibiting a high degree of uncertainty. Obviously it is possible to reduce uncertainty about some variables by gathering information before migrating. But certainly there remains a rest of uncertainty which cannot be reduced any more. Consider e.g. the "quality of life" in a foreign country. It is impossible to know it before living in the country for at least some time. In other words learning about quality of life in a foreign country means "learning by doing".

At a first glance one could think to extend *Sjaastad*'s model by admitting the net returns $N_j(t) = (R_j(t) - C_j(t))$ to be random variables with a given subjective probability distribution. Then one could calculate expected capital values $E(K_j)$ for each country and apply rule I analogously. But in most cases the migrant will detect some discrepancies between his expectations and his real experiences after living for some periods in a foreign country. According to rule I he now has to start his decision procedure anew with revised expectations.

Naturally a rational decision maker will take into account later revisions of his decision *now*. As rule I is not flexible enough to deal with this kind of problems we need a sequential decision model for migration. In the following section we will introduce such a sequential model which especially can treat the problem of incomplete information adequately. The mathematical structure of the model has been adapted from the work of *Mandelbaum* (1986), *Varaiya* et al. (1984), *Gittins* (1979) and his "school". *J. McCall/B. McCall* (1984) were the first who emphasized the relevance of these formal models for the migration problem. Our paper is an extension of *McCall*'s framework to a more general stochastic environment.

II. A sequential model of international migration

1. *The formal model*

Our formal model will be introduced in three parts. We will start with the *return and information structure* of the migration process.

We suppose that a migrant has to choose to go into N countries (indexed by j) each characterized by a stochastic process of net returns on migration

$N_j := \{N_j(t)\}_{t=1}^{\infty}$

where $N_j(t)(:= (R_j(t) - C_j(t))$ is a real valued random variable[4]

$N_j(t) : (\Omega, F, P) \rightarrow (\mathbb{R}, B)$

which is interpreted as the net return on migration in country j after living there t-times. Furthermore each country is characterized by an information process

$F_j = \{F_j(t)\}_{t=0}^{\infty}$

where $F_j(t) \subseteq F$ (with $F_j(t) \subseteq F_j(t+1)$, $\forall\, t$) has to be interpreted as the information the migrant has about country j after living there t-times. For the following we need two formal assumptions concerning the concepts introduced above:

α) $N_j(t)$ is bounded and $F_j(t-1)$ – measurable,

β) F_j^{∞} are independent σ-fields, where F_j^{∞} denotes the σ-field that is generated by the σ-algebras $F_j(t)$ $(t \geq 0)$.

According to α) N_j is a predictable process. Assumption β) implies that net rewards in different countries are stochastically independent. We don't need any specific assumption concerning stochastic dependence of net returns within the same country. This is an extension of Gittins former work where $\{N_j(t)\}$ was supposed to be a Markovian process.

Another important concept is the *migration strategy* S which is from a mathematical point of view a stochastic process

$S = \{S(t)\}_{t=0}^{\infty}$

with

$S(t) : (\Omega, F, P) \rightarrow \tilde{N} := \{(n_1, \ldots, n_N) \mid n_j \in N\}.$

If a state $\omega \in \Omega$ occurred, the vector

$S(t)(\omega) = (n_1, \ldots, n_N)$

has to be interpreted as follows: After t periods the migrant lived n_1-times in country 1, n_2-times in country 2, … and n_N-times in country N $(\sum_1^N n_j = t)$. In

[4] (Ω, F, P) denotes the underlying probability space and B denotes the σ-algebra of Borel-subsets of $\mathbb{R}$.

other words a migration strategy counts the number of living periods in each country. Furthermore we will suppose that a migrant can live during one period in one country only and that he has to base his migration decision on the total information available for him at the time point in question. We will express these requirements by the following assumptions.

γ) $S(0) = (0, \ldots, 0) \in \tilde{N}$,

δ) for all $t \geqslant 1$

$S(t) - S(t-1) = e_j = (0, \ldots, 1, \ldots, 0)$ for some $j = 1, \ldots, N$,

ε) for all $t \geqslant 1$

$S(t)$ is measurable with respect to $F_{S(t-1)}$, that is the σ-algebra of all events that occurred "prior"[5] to $S(t-1)$.

Finally, we have to specify the *pay-off* a migrant will obtain from his migration strategy. Given a strategy S we can define the net-return at the t-th period as follows

$$N_t(S) := \sum_1^N N_j(S_j(t))\,(S_j(t) - S_j(t-1))$$

where $S_j(t)$ denotes the j-th component of the vector $S(t)$. It follows from this definition that a migrant obtains a net return only from the country where he is living at that period. Obviously this is a reasonable assumption that corresponds to our assumption δ) above.

The total discounted net benefit from migration strategy S then is given by the random variable

$$N(S) := \sum_1^\infty \beta^{t-1} N_t(S)$$

where $\beta \in (0, 1)$ denotes the discount rate. The goal of the migrant is to find a migration strategy that maximizes $E(N(S))$, the expected present value of net migration benefits. In other words, we assume risk-neutrality.

[5] This is an extension of the corresponding concepts of traditional theory of Optimal Stopping to Multiparameter stopping processes (for details see *Mandelbaum/Vanderbei* (1981), *Krengel/Sucheston* (1981)).

Obviously this is a non-trivial sequential decision problem which is difficult to solve without any restrictive assumptions. Fortunately there has been developed a solution method by which it is possible to get a very good insight into the structure of optimal migration policies. This method has been introduced at first by *Gittins* (see *Gittins/Jones* (1974), *Gittins* (1979), *Whittle* (1980)) approximately ten years ago. We will discuss a general version of the method in the following chapter.

2. *The Gittins-Index Method*

It can be demonstrated easily that there exists at least one maximizing strategy for the migration problem (see *Mandelbaum* (1986)). For economic applications it is more interesting to see how the optimal solution can be characterized. *Gittins* was able to prove that the complicated decision problem may be solved by solving N simpler stopping problems at each decision point.

More precisely let there be given a vector $(n_1, \ldots, n_N) \in \tilde{N}$ and a collection of information σ-algebras $\{F_j(n_j)\}_{j=1}^{N}$ describing a migrant's migration history and his state of information after $t = \sum_{j=1}^{N} n_j$ periods then a migrant has to solve the following stopping problem separately for each region:

– Calculate a stopping time $\tau \geqslant (n_j + 1)$ such that the random variable

$$P_j(\tau) := \frac{E(\sum_{n_j+1}^{\tau} \beta^t N_j(t) \mid F_j(n_j))}{E(\sum_{n_j+1}^{\tau} \beta^t \mid F_j(n_j))}$$

is made as large as possible. This largest value will be called *Gittins index* denoted by $v_j(n_j)$ and obviously is formally given by

$$v_j(n_j) = \operatorname*{ess\,sup}_{\tau \geqslant (n_j+1)} P_j(\tau).$$

One should not confuse $P_j(\tau)$ with the concept of "(expected) profit per period". Even though both concepts are similar $P_j(\tau)$ differs from per period profit with respect to the fact that one has to divide discounted profits over a given number of periods by the discounted number of these periods. Therefore we will call $P_j(\tau)$ quasi-per period profit.

The *index field* is the multiparameter process $v = \{v(\tilde{n}),\ \tilde{n} \in \tilde{N}\}$ defined by

$$v(\tilde{n}) = \max_{j \in \{1, \ldots, N\}} v_j(n_j) \quad (\tilde{n} = (n_1, \ldots, n_N)).$$

With the help of the concept of an index field we are now able to formulate the main theorem concerning the solution of the migration problem.

Theorem (*Mandelbaum* (1986)): The class of optimal strategies coincides with the class of strategies S such that

$$v(S(t)) = v_j(S_j(t)) \quad \text{on} \quad \{S(t+1) - S(t) = e_j\}.$$

for all $t = 0, 1, 2, \ldots$

In other words a migration strategy is optimal if a decision maker always lives in the country with the largest Gittins index. Now we can summarize the optimal migration strategy by the following rule.

Rule II: Suppose there are N countries, each characterized by a net return and information process $\{N_j;\ F_j\}_{j=1}^{N}$. Suppose furthermore that there is given a vector $(n_1, \ldots, n_N) \in \tilde{N}$ and a collection of σ-fields $\{F_j(n_j)\}_{j=1}^{N}$, then choose country j^* where

$$v_j^*(n_j^*) = \max_{j \in \{1, \ldots, N\}} v_j(n_j).$$

With the help of the following lemma, which gives a nice characterization of the optimal stopping time τ we can derive an interesting interpretation of rule II.

Lemma 1 (*Mandelbaum* (1986)): The essential supremum of $P_j(\tau)$ is attained by the stopping time

$$\tau(n_j) = \inf\{t \geqslant (n_j + 1) \mid v_j(t) \leqslant v_j(n_j)\}.$$

Now suppose a migrant is at the beginning of his migration process. According to rule II together with the above lemma the optimal migration strategy then can be characterized as follows:

- At first calculate $v_j(0)$ $(j = 1, \ldots, N)$ and go into the country with the largest index, here denoted by j',
- stay in the country at least for $\tau(0)$ periods where $\tau(0)$ is given by the lemma. Then calculate $v_{j'}(\tau(0))$ again,
- if $v_{j'}(\tau(0)) < v_j(0)$ for at least one j then move to another country (with largest $v_j(0)$) otherwise stay in country j', calculate a stopping time $\tau(\tau(0))$

such that

$$P_{j'}(\tau(\tau(0))) = \underset{\tau \geqslant (\tau(0)+1)}{\operatorname{ess\,sup}} P_{j'}(\tau)$$

and stay again for at least $(\tau(\tau(0)) - \tau(0))$ periods in j'. Then calculate the Gittins index for country j' anew ....

In other words a migrant needs to calculate only one Gittins index at some time points determined endogenously by the information gathered by the migrant. Therefore we can say that by the stopping times determined above a (preliminary) *staying strategy* can be derived such that the migrant calculates the Gittins index anew provided the end of the (preliminary) staying period is reached.

The proof of the Gittins-index theorem is not trivial. Fortunately a lucid proof of the theorem can be given for the simplified case of a deterministic net-return process. This will be done in the following chapter.

3. *Validity of the Gittins-index rule*

We want to illustrate the main idea of the proof of the theorem by an illustrative example. Suppose there are two countries with the deterministic net-return processes

$$N_1 = \{X(1), X(2), \dots\},$$

$$N_2 = \{Y(1), Y(2), \dots\}, \quad \text{where } X(t),\ Y(t) \in \mathbb{R}_+.$$

The Gittins indices then are given by

$$\nu_1(0) = \max_{\tau \geqslant 1} \frac{\sum_1^\tau \beta^t X(t)}{\sum_1^\tau \beta^t}$$

$$\nu_2(0) = \max_{\tau \geqslant 1} \frac{\sum_1^\tau \beta^t Y(t)}{\sum_1^\tau \beta^t}.$$

Now suppose that τ^* is the solution of the stopping problem for country 1, that is

$$\nu_1(0) = P_1(\tau^*) = \max_{\tau \geqslant 1} P_1(\tau).$$

Furthermore we suppose $\nu_1(0) \geqslant \nu_2(0)$.

Under these assumptions we can prove the following lemma.

Lemma 2

$$\frac{\sum_s^{\tau^*} \beta^t X(t)}{\sum_s^{\tau^*} \beta^t} \geqslant v_1(0), \qquad 1 \leqslant s \leqslant \tau^* \tag{α}$$

$$\frac{\sum_1^s \beta^t Y(t)}{\sum_1^s \beta^t} \geqslant v_2(0), \qquad s \geqslant 1. \tag{β}$$

Proof (β) follows immediately from the definition of $v_2(0)$.

$$(\alpha): \ v_1(0) \geqslant \frac{\sum_1^s \beta^t X(t)}{\sum_1^s \beta^t} = \frac{\sum_1^{\tau^*} \beta^t X(t) - \sum_{s+1}^{\tau^*} \beta^t X(t)}{\sum_1^{\tau^*} \beta^t - \sum_{s+1}^{\tau^*} \beta^t},$$

which implies

$$\sum_1^{\tau^*} \beta^t v_1(0) - \sum_{s+1}^{\tau^*} \beta^t v_1(0) \geqslant \sum_1^{\tau^*} \beta^t X(t) - \sum_{s+1}^{\tau^*} \beta^t X(t). \tag{*}$$

Because of $\sum_1^{\tau^*} \beta^t v_1(0) = \sum_1^{\tau^*} \beta^t X(t)$ we have from inequality(*)

$$\sum_1^{\tau^*} \beta^t X(t) + \sum_{s+1}^{\tau^*} \beta^t X(t) \geqslant \sum_1^{\tau^*} \beta^t X(t) + \sum_{s+1}^{\tau^*} \beta^t v_1(0),$$

which implies (α).

Now let us consider the sequence of net returns by using an arbitrary strategy S. This sequence will be an interweaving of the two sequences N_1 and N_2, which will be denoted in the sequel by

$$Z(1), \ Z(2), \ Z(3), \dots .$$

Furthermore let T be the time when S chooses country 1 for the τ^*-th time so that $Z(T) = X(\tau^*)$. Then the Z sequence must be of the form

$$Y(1), \dots, Y(k_1), \ X(1), Y(k_1+1), \dots, Y(k_2), \ X(2), \dots, Y(k_{\tau^*}),$$
$$X(\tau^*), Z(T+1), Z(T+2), \dots .$$

Next consider a strategy $\tilde{S}$ which requires living in country 1 for the first τ^* periods, in country 2 for k_{τ^*} times and then follows strategy S, which yields the net return sequence

$$X(1), \dots, X(\tau^*), \quad Y(1), \dots, Y(k_{\tau^*}), \quad Z(T+1), Z(T+2), \dots .$$

Then we can compute

$$N(S) = \sum_1^{k_1} \beta^t Y(t) + \cdots + \beta^{\tau^*-1} \sum_{k_{\tau^*}-1}^{k_{\tau^*}} \beta^t Y(t) + \sum_1^{\tau^*} \beta^{k_t+t} X(t) + \sum_{T+1}^{\infty} \beta^t Z(t),$$

$$N(\tilde{S}) = \sum_1^{\tau^*} \beta^t X(t) + \beta^{\tau^*} \sum_1^{k_{\tau^*}} \beta^t Y(t) + \sum_{T+1}^{\infty} \beta^t Z(t).$$

By switching from S to $\tilde{S}$ we see that we gain a positive amount by getting $X(t)$ earlier. This gain will be denoted by Δ_x where

$$\Delta_x := \sum_1^{\tau^*} (1-\beta^{k_t}) \beta^t X(t).$$

On the other hand we loose by postponing the net returns $Y(t)$ by an amount Δ_y which is defined by

$$\Delta_y := (1-\beta^{\tau^*}) \sum_1^{k_1} \beta^t Y(t) + (\beta - \beta^{\tau^*}) \sum_{k_1+1}^{k_2} \beta^t Y(t) + \cdots$$

$$+ (\beta^{\tau^*-1} - \beta^{\tau^*}) \sum_{k_{\tau^*}-1}^{k_{\tau^*}} \beta^t Y(t).$$

The main step now is to show that $\Delta_x \geqslant \Delta_y$ which obviously implies

$$N(\tilde{S}) - N(S) = \Delta_x - \Delta_y \geqslant 0. \qquad (*)$$

Thus it is better to follow rule II until time τ^*. The argument can now be repeated starting at time τ. This proves the optimality of the index rule for our simple example.

To prove the required inequality we show that Δ_x and Δ_y are bounded below resp. above.

Lemma 3 If $\nu_1(0) \geqslant \nu_2(0)$ then we have

$$\Delta_x \geqslant \nu_1(0) \sum_1^{\tau^*} (1-\beta^{k_t})\beta^t$$

and

$$\Delta_Y \leqslant \nu_1(0) \sum_1^{\tau^*} (1-\beta^{k_t})\beta^t.$$

Proof By some simple transformations we obtain

$$\Delta_x = \sum_1^{\tau^*} (\beta^{k_{t-1}} - \beta^{k_t}) \sum_{n=t}^{\tau^*} \beta^n X(n)$$

$$\geqslant \nu_1(0) \sum_1^{\tau^*} (\beta^{k_{t-1}} - \beta^{k_t}) \sum_{n=t}^{\tau^*} \beta^n \qquad \text{(by Lemma } 2\alpha))$$

$$= \nu_1(0) \sum_1^{\tau^*} (1-\beta^{k_t})\beta^t,$$

and

$$\Delta_y = \sum_1^{\tau^*} (\beta^{t-1} - \beta^t) \sum_{n=1}^{k^t} \beta^n Y(n)$$

$$\leqslant \nu_1(0) \sum_1^{\tau^*} (\beta^{t-1} - \beta^t) \sum_{n=1}^{k_t} \beta^n \qquad \text{(because of } \nu_1(0) \geqslant \nu_2(0) \text{ and lemma } 2\beta))$$

$$= \nu_1(0) \sum_1^{\tau^*} (1-\beta^{k_t})\beta^t.$$

Obviously lemma 3 implies that the gain from "consuming" $X(t)$ earlier overcompensates the loss induced by postponing the "consumption" of $Y(t)$. The validity of inequality (*) now immediately follows from lemma 3.

III. Economic Implications

In the last section we presented a formal microeconomic model of the migration decision by which it is especially possible to treat the problem of incomplete information adequately. Furthermore we showed that there exists a solution method for the decision problem (= Gittins-index method) which allows an insightful interpretation of the structure of optimal migration decisions.

In this section we want to demonstrate furthermore that the Gittins-index theory also simplifies the comparative statics of the migration problem considerably. We will demonstrate this by a highly stylized example for guest worker migration. It should be kept in mind here that all results of the example can be generalized to a more realistic setting. This will be demonstrated elsewhere (*Berninghaus/Seifert-Vogt* (1987)).

1. *A simple example*

Now let us consider a guest worker who has to choose to go into a host country ($j=1$) or to stay at home ($j=2$). If he stays at home he is supposed to be completely informed about his net returns

$$N_2(t) = (w_2 + \alpha_2) \qquad \text{for all } t,$$

where w_2 denotes the real wage per period the worker can earn at home (= pecuniary returns of migration) and α_2 the quality of life in the home country (= non-pecuniary returns).

Furthermore the guest worker is supposed to be incompletely informed about the non-pecuniary returns in the host country. But he will obtain full information exatly after one period living in the host country. To formalize this assumption we suppose that the non-pecuniary return for the worker before migration is a random variable V where

$$V = \begin{cases} \alpha_1 & \text{with probability } 1/2 \qquad (\alpha_1 > 0), \\ -\alpha_1 & \text{with probability } 1/2. \end{cases}$$

On the other hand the worker is supposed to be completely informed about the pecuniary returns w_1 (= real wage per period) in the host country. This may be motivated by the assumption that the worker can sign his contract with a fixed wage rate before migrating. The net return process N_1 then is supposed to be a Markov process where

$$N_1(1) = (w_1 - C_1(1)) \quad (\text{where } C_1(1)(1-\beta) < \alpha_1)$$

$$N_1(2) = (w_2 + V)$$

and

$$Pr\{N_1(t+1) = w_2 \pm \alpha_2 | N_1(t) = w_2 \pm \alpha_2\} = 1 \qquad \text{for all } t \geqslant 2.$$

In other words the state $(w_2 \pm \alpha_2)$ is an absorbing state of the Markov process. The formal model given above obviously corresponds to the verbal descriptions of the example. After migrating to the host land bearing migration costs $C_1(1)$ the migrant will be completely informed about the realization of the random variable V.
It is easy to calculate the Gittins indices for this simple example.

Fact 1 The Gittins indices are given by

$$v_1(0) = \frac{(w_1 - C_1(1))\frac{1}{2} + \frac{\beta}{1-\beta}(w_1 + \alpha_1)\frac{1}{2}}{\frac{1}{2} + \left(\frac{1}{1-\beta}\right)\frac{1}{2}}$$

$$v_1(n_1) = (w_1 \pm \alpha_1) \qquad \text{for all } n_1 \geqslant 1,$$

$$v_2(n_2) = w_2 + \alpha_2 \qquad \text{for all } n_2 \geqslant 0.$$

As the net returns per period in country 2 are constant we have[6]

$$v_2(0) = (w_2 + \alpha_2) = \max_{\tau \geqslant 1} \frac{\sum_1^\tau \beta^t N_2(t)}{\sum_1^\tau \beta^t}$$

$$= (w_2 + \alpha) \max_{n \geqslant 1} \frac{\sum_1^n \beta^t}{\sum_1^n \beta^t} = (w_2 + \alpha_2)$$

$$= v_2(1) = v_2(2) = \cdots .$$

The same reasoning is valid for $v_1(n_1)$ if $n_1 \geqslant 1$. Concerning the computation of $v_1(0)$ we can see that $v_1(0)$ results from the following stopping strategy:

- Stay for one period and stop if $(-\alpha_2)$ has realized
- stay for one period and don't stop if $(+\alpha_2)$ has realized
  $\rightarrow$ expected net returns: $(w_1 - C_1)\ 1/2 + (\beta/(1-\beta))(w_2 + \alpha)\ 1/2$.

[6] Obviously in this case we need to consider constant stopping times $\tau(\equiv n;\ n \in N)$ only.

It is easy to see hat any other stopping strategy will give an inferior result.

By the indices $\nu_j(.)$ it is possible now to derive the migration history of the migrant in our stylized example. Suppose $\nu_1(0) < \nu_2(0)$, then migration will not take place at all. If $\nu_1(0) > \nu_2(0)$, then according to rule II the worker will move for at least one period into the host country. After one period living there the worker will be completely informed about V. Then two inequalities are possible:[7]

- $\nu_1(1) \geqslant \nu_2(0) \rightarrow$ the worker will stay in the host country forever,
- $\nu_1(1) < \nu_2(0) \rightarrow$ the worker will return to his home country.

Here we have an insightful example for explaining remigration by a satisfactory sequential model.

2. *Comparative statics*

Now it remains to be analyzed how the optimal migration strategy will have to be changed if some selected exogenous parameter of the decision problem are changed. In doing this we will utilize the results of our example described in the previous chapter. Needless to say that all comparative static results can be derived in a much more general model[8].

According to the simplicity of our example we have to consider only three relevant exogenous parameters ($C_1(1)$, w_j, α_j). As the Gittins indices here are given explicitly the comparative static results are obtained simply by taking partial derivatives. In more general models where the Gittins index sometimes is given only implicitly (via a functional equation) one has to apply different methods.

In detail we have the following results which seem to be plausible.

Fact 2

$$\frac{\partial \nu_1(0)}{\partial C_1(1)} < 0, \quad \frac{\partial \nu_1(0)}{\partial w_1} > 0,$$

$$\frac{\partial \nu_2(0)}{\partial w_2} > 0, \quad \frac{\partial \nu_2(0)}{\partial \alpha_2} > 0,$$

[7] In case $\nu_1(0) = \nu_2(0)$ any tie breaking rule is applicable.
[8] See a forthcoming paper by *Berninghaus/Seifert-Vogt* (1987).

In summary one can conclude that increasing migration costs deter migration while increasing returns make migration more attractive. Special attention deserves the variation in α_1, the realized value of the random variable V. Here we have again

$$\frac{\partial v_1(0)}{\partial \alpha_1} > 0.$$

But the interpretation in this case is more delicate than before. An increase in α_1 results simultaneously in a symmetric decrease of $(-\alpha_1)$. As the mean value of the new random variable denoted by V' has not changed the new distribution of the non-pecuniary returns in country 1 exhibits "more risk" according to the "mean preserving spread" ordering (see *Rotschild/Stiglitz* (1970)). Consequently we obtain the following result.

Fact 3 If the distribution of V in a country j is ranked above the distribution of V' in another country j' according to the criterion of "mean preserving spread" (that is if V' is "more risky" than V) then (ceteris paribus) country j' is more attractive for migration than country j.

From Fact 3 we are able to deduce another interesting interpretation of the comparative static result concerning variations of α_1. In many microeconomic models of decision making under incomplete information one usually formalizes a decision maker's knowledge of the value of an unknown variable (e.g. his non-pecuniary net return) as follows: The decision maker presupposes that there exists an upper bound and a lower bound $(\underline{\alpha}_1, \overline{\alpha}_1)$ for the value of the variable and that the variable is uniformly distributed over the interval $[\underline{\alpha}_1, \overline{\alpha}_1]$ (*see fig.* 1).

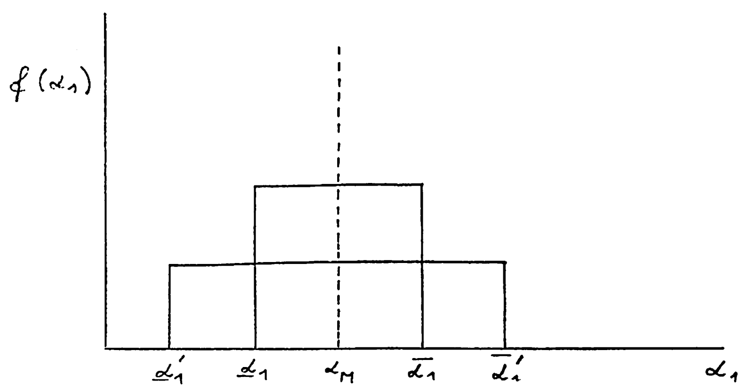

Fig. 1
Density function $f(.)$ for α_1

To be "less informed" may then be formalized by enlarging the inverval $[\underline{\alpha}_1, \overline{\alpha}_1]$ symmetrically to $[\underline{\alpha}_1', \overline{\alpha}_1']$ *where* $\underline{\alpha}_1' = \underline{\alpha}_1 - \varepsilon$, $\overline{\alpha}_1' = \overline{\alpha}_1 + \varepsilon$ $(\varepsilon > 0)$. Obviously the new distribution of the variable differs from the former one by a larger mean preserving spread (mass has been shifted to the tails without altering the expected value) and therefore we can apply here the result of fact 3 analogously[9] to obtain

Fact 4 If a migrant is less informed (in the sense defined above) about the values of some specified variables in country j than in country j' he will (ceteris paribus) prefer migrating to country j.

This is a rather surprising result which might shed new light on the relevance of imcomplete information as a determinant of migration. In the migration literature one takes distance between countries as a proxy for information of a migrant. Together with the empirically highly supported result that increasing distance has a deterring effect on migration many authors conclude that decreasing information (= increasing distance) must have a deterring effect on migration. In contrast to this view we demonstrated above that this cannot be true for risk-neutral rational decision makers. Consequently one should reconsider taking distance as a proxy for incomplete information in migration models.

For a risk-neutral migrant the intuition behind the result of Facts 3 and 4 respectively is clear. A distribution exhibiting more risk gives the migrant a chance of getting higher net returns (or a higher chance of getting high net returns) together with a chance of getting lower net returns (or a higher chance of getting lower net returns). As the migrant can revise his migration decision in each period if low net returns have realized and as he is supposed to be risk neutral he will prefer to try out a country with more risk provided the countries are equally characterized in all other respects.

Conclusion

In the previous section we presented a formal model of international migration by which it is possible to deal with the problem of incomplete information satisfactorily. Even though our model is still rather simple it already gives some insight into the effects of incomplete information on migration decisions. Therefore we can hope to get a better understanding of some relevant (empirically supported) phenomena as remigration, increasing willingness for further migra-

[9] A rigorous proof of the result can be found in *Berninghaus/Seifert-Vogt* (1987).

tion after moving for the first time etc. It doesn't make much sense to treat these phenomena by the traditional Human Capital approach. A better explanation can be given within our sequential decision model framework.

For example one can consider remigration of Greek guest workers which has created serious problems for the Greek economy (*Katseli/Glytsos* (1986)) during 1965 to 1975. At that time the average stay of a Greek guest worker in Germany was 3 years. In analyzing the reasons for remigration health and family problems turned out to be the most important ones. By interpreting the state of health of a migrant as an important contribution to the non-pecuniary benefits in the host country our simplified model in III.1. can be applied at once. Obviously the quality of life experiment for the remigrating guest workers had negative results. Complete information about the non-pecuniary benefits obviously came in after 2 years living in the host country. A result that fits remarkably well into the framework of our simple example in III.1.

Naturally much work remains to be done to generalize our model. Here we want to emphasize only few points. At first one should analyse the optimal migration policy of a risk-averse migrant. Clearly for a risk-averse individual the results in Facts 3 and 4 would have to be modified. But we conjecture that they might remain valid for decision makers who are only "mildly" risk-averse. In this context one should also try to integrate the dynamic choice theory of *Kreps/Porteus* (1978) into our decision problem. For a migrant will especially be interested in the time period of resolution of uncertainty (about pecuniary or non-pecuniary variables in a country). And this aspect of a decision problem under incomplete information has been analyzed by *Kreps/Porteus*.

In this context one could also consider to introduce time dependent discount-factors which are furthermore different for discounting gross returns and for discounting costs of migration. By this procedure one could express that the migrant might be more interested in resolving uncertainty about returns than about costs or vice versa.

Another point for further research is the analysis of so-called "super-processes" (see*Gittins* 1979) which are special sequential decision processes with the following additional complication: A decision maker has to solve in each period an optimization problem such that the result of this problem influences the per period pay-off and the net return process in the future. As a straight-forward application of this concept to the migration problem we can think of a migrant who has the option to make saving decisions in the foreign country. Obviously such saving decisions will alter the stream of present and future net returns of migration. Another important application is information gathering during each period. Instead of assuming a fixed stream of incoming information $\{F_j(t)\}$ for each country one could suppose that a migrant wants to control the information coming in in each period. Naturally information gathering should not be costless. As the result of information gathering will influence the state of information for the following period this modified migration problem can also be regarded as a particular bandit super-process. Unfortunately very little is known about super-processes, especially the Gittins-index policy for super-processes is only valid under additional restrictions (see *Glazebrook* (1982), *Whittle* (1980), *Varaiya* et al (1984)). Further research is necessary to obtain results that are applicable to our migration problem.

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Abstract

International Migration under Incomplete Information

We present a model for deriving optimal sequential migration decisions under incomplete information. As a main characteristic of the model, the decision maker can rationally learn about pecuniary and non-pecuniary net return on migration in different countries by migrating into these countries. Furthermore it is shown that the optimal migration policy can be characterized by a "simple" procedure, called "Gittins-index policy". In the context of our sequential migration model we find more satisfying explanations for some relevant empirical observations (e.g. remigration) than it would be possible within the traditional deterministic approach.

Zusammenfassung

Migrationsentscheidungen bei unvollständiger Information

Im vorliegenden Artikel wird ein Entscheidungsmodell für optimale, sequentielle Migrationsentscheidungen bei unvollständiger Information entwickelt. Charakteristisch für das Modell ist die Möglichkeit "rationalen Lernens" über die pekuniären und nicht-pekuniären Netto-Erträge der Wanderung in verschiedene Länder. Es wird gezeigt, dass die optimale Migrationspolitik durch ein "einfaches" Lösungsverfahren, die sog. Gittins-Index-Methode, charakterisiert werden kann. Im Rahmen unseres sequentiellen Migrationsmodells ist es möglich, viele empirisch relevante Phänomene (z.B. Rückwanderung) befriedigender als mit einem der bisher bekannten traditionellen, deterministischen Ansätze zu erklären.

Résumé

Migration sous information incomplète

Cet article présente un modèle pour la dérivation de décisions optimales de migration séquentielle sous information incomplète. Comme particularité essentielle, le modèle offre la possibilité "d'apprendre rationnellement" les rendements nets – pécuniaires et non-pécuniaires – de la migration dans des pays divers. De plus, il est montré que la politique optimale de migration peut être caractérisée par un procédé "simple", appelé "la méthode de l'indice Gittins". Dans le cadre de notre modèle de migration séquentielle, il est possible de trouver des explications plus satisfaisantes de beaucoup de phénomènes empiriques relevants (comme par exemple remigration) que celles trouvées par un rapprochement déterministe traditionnel.