Term Premium and Volatility: A Nonlinear Analysis of the Swiss Interest Rates

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1. INTRODUCTION

The rejection of the constant term premium version of the Expectations Hypothesis of the Term Structure (EHTS) has been documented in numerous studies and has spawned a second generation of papers attempting to account for it. One of the most influential explanations of the negative results has been to attribute the empirical failure to a time varying term premium. The ARCH-in-mean model introduced by ENGLE, LILIEN and ROBINS (1987) is perhaps the most popular model attempting to tie the term premium to a time varying conditional variance. Using six over three month and two over one month U.S. treasury bills they find significant evidence in favor of the EHTS. Studies that apply the ARCH-in-mean model to other data sets, however, find little evidence that ARCH-in-mean helps to explain the rejection of the EHTS. Among others, KUGLER (1994) uses three over one month Euro interest rates for 9 countries and finds that, with the exception of Japan, the EHTS has to be rejected for all other 8 countries even taking into account of the ARCH-in-mean effect. In addition, by employing nonparametric (kernel) regression to explore the regression test results in a number of Canadian and US data set, CAMPBELL and GALBRAITH (1995) find that incorporating ARCH-in-mean in models of the term premium is not sufficient to account for rejections of the expectations hypothesis.

Moreover, as pointed out by PAGAN and ULLAH (1988), consistent estimation in the ARCH-in-mean model requires the full model to be correctly specified. Any misspecification in the variance equation generally leads to a biased and inconsistent estimate of the parameters in the mean equation. Further evidence of the sensitivity of the parameter estimate in the ARCH-in-mean model with respect to different model specifications is given in BOLLERSLEV, CHOU and KRONER (1992).

In this paper, we ask whether the empirical results found by incorporating the ARCH-in-mean effect in modeling the term premium can be reproduced by employing a more general, nonparametric approach. We first investigate whether the ARCH-in-

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mean type models provide sufficient nonlinearities for the explanation of the rejection of the EHTS. In order to avoid bias and spurious results due to a misspecification, we then undertake a nonlinear impulse response analysis by utilizing the method developed by GALLANT, ROSSI and TAUCHEN (1993). This nonparametric estimation strategy allows us to approximate the unknown conditional variance function by a series expansion so that any specification error can be avoided. The data set we use here is the weekly data for one and three months Euro Swiss franc interest rates over the period from 1978 to 1992.

The remaining part of the paper is organized as follows. Section 2 contains empirical results for the standard test of the expectations hypothesis as well as results on the conditional means volatility relationship of Swiss interest rates using EGARCH-in-mean model. Section 3 provides a brief introduction to the nonlinear impulse response analysis and presents the empirical results obtained with our data. Section 4 concludes this paper.

2. EXPECTATIONS HYPOTHESIS OF THE TERM STRUCTURE AND ARCH-IN-MEAN MODELS: A PRELIMINARY EXPLORATION

The usual framework of analysis for the term structure of interest rates is the so-called expectations hypothesis. It relates the long rate to the current and expected future short rate and a constant term premium. In our application to weekly three and one month interest rates data the corresponding equation is

$$R_t = \frac{1}{3}(r_t + r^e_{t+4} + r^e_{t+8}) + \theta \tag{1}$$

where θ is the term premium.

This equation can be rewritten as

$$\begin{aligned} R_t - r_t &= \frac{1}{3}(r^e_{t+4} - r_t + r^e_{t+8} - r_t) + \theta \\ &= \frac{1}{3}\Big(\sum_{j=1}^{4} 2(r^e_{t+j} - r^e_{t+j-1}) + (r^e_{t+4+j} - r^e_{t+4+j-1}) + \theta\,. \end{aligned}$$

Therefore, changes in the spread are, according to this model, caused by changes in the expected future short rate. In addition, under the usual $I(1)$ assumption for r_t, the model implies an $I(0)$ property for $R_t - r_t$, i.e. R_t and r_t are cointegrated. Replacing the expected by observed values we obtain the following testing equation:

$$y_t \equiv \sum_{j=1}^{4}(2\Delta r_{t+j} + \Delta r_{t+j+4}) = \alpha + \beta(R_t - r_t) + e_{t+8}\,, \tag{2}$$

with $\alpha = -3\theta$, $\beta = 3$ and e_{t+8} a MA(7) error term reflecting expectational errors for Δr_t up to eight steps ahead:

$$e_{t+8} = 2\sum_{j=1}^{4}\varepsilon_{t+j} + \sum_{j=1}^{4}\varepsilon_{t+j+4}\,.$$

Hence, the above model implies that the spread has predictive power for the short rate changes. Moreover, under rational expectations this equation can be consistently estimated by OLS. The covariance matrix estimate has to be, of course, corrected for MA(7) autocorrelation, and the heteroscedasticity also has to be accounted for given the ARCH effects expected for (weekly) interest rate data.

Table 1 contains the result of this standard test of the expectations theory for our weekly Euro Swiss franc data over the period from the beginning of 1978 to mid 1992. For this data set the spread has statistically significant predictive power for the short rate change, but the corresponding slope coefficient is clearly smaller than 3, the value implied by the expectations theory. These results are in line with the findings of earlier studies using monthly Euro market Sfr. rates (e.g. KUGLER, 1994).

Of course, the rejection of the expectations hypothesis could be caused by a time variant term premium. In order to implement this idea we adopt an EGARCH-in-mean approach and replace $\alpha + e_{t+8}$ in equation (2) by $\alpha + \gamma \log\sigma^2_{t+1} + \varepsilon_{t+8} + \sum_{i=1} c_i\, \varepsilon_{t+8-i}$, the test equation becomes

$$y_t = \alpha + \beta(R_t - r_t) + \gamma \log\sigma^2_{t+1} + \varepsilon_{t+8} + \sum_{i=1}^{7} c_i\, \varepsilon_{t+8-i}\,, \tag{3}$$

where

$$\log \sigma^2_{t+1} = a_0 + a_1 z_t + a_2|z_t - \sqrt{\tfrac{2}{\pi}}| + b \log \sigma^2_t\,, \tag{4}$$

with $z_t \equiv \varepsilon_t / \sigma_t$, $z_t \sim INI(0,1)$.

The EGARCH-in-mean model described in equation (3) and (4) has two essential features. First, it allows for an influence of the volatility of the innovation in the short rate on the term premium. The coefficient would be negative as the term premium enters equation (2) with a negative sign. Second, modeling the volatility dynamics as an

Table 1:
Tests of the Expectations Hypothesis of the Term Structure of Interest Rates with and without Volatility Dependent Term Premium. Three and one Months Euro Sfr. Rates, Weekly Data 1978-92

$$2\sum_{j=1}^{4}\Delta r_{t+j} + \sum_{j=1}^{4}\Delta r_{t+j+4} = \alpha + \beta(R_t - r_t) + \gamma \log \sigma_{t+1}^2 + \varepsilon_{t+8} + \sum_{i=l}^{7} c_i \varepsilon_{t+8-i}$$

$$\log \sigma_{t+1}^2 = a_0 + a_1 z_t + a_2 |z_t - \sqrt{\frac{2}{\pi}}| + b \log \sigma_t^2$$

	OLS	E-GARCH-M
α	-0.275 (0.145)	-0.400 (0.171)
β	1.87 (0.412)	2.11 (0.121)
χ	–	-0.149 (0.103)
c_1	–	0.893 (0.038)
c_2	–	0.797 (0.046)
c_3	–	0.710 (0.053)
c_4	–	0.609 (0.054)
c_5	–	0.511 (0.052)
c_6	–	0.455 (0.044)
c_7	–	0.355 (0.036)
a_0	–	-0.0169 (0.011)
a_1	–	0.00086 (0.026)
a_2	–	0.319 (0.052)
b	–	0.972 (0.011)
$Q_z(24)$	–	35.0
$Q_{zz}(24)$	–	35.1
z-Skewness	–	-0.034
z-Kurtosis	–	3.24

Estimated standard errors in parentheses.

exponential GARCH, as introduced by NELSON (1990), allows for asymmetric effects with respect to the size and sign ($a_1 \neq 0$) of shocks on volatility.

The Maximum Likelihood estimates for this model are given in the third column of Table 1. The slope coefficient estimate ends up closer to 3, which is the value under the null hypothesis. However, it is significantly different from 3. The coefficient estimate for the conditional variance term $\hat{\gamma}$ is negative, but hardly significantly different from zero. Furthermore, the conditional variance is highly persistent ($\hat{b} = 0.972$). However, there is no indication of asymmetric responses with respect to different signs of the shocks ($a_1 = 0.00086$). Thus, introducing a volatility dependent term premium brings the results closer to the implications of the expectations hypothesis, but the effect of the conditional variance on the conditional mean is rather weak. Additionally, the Q-statistics for the standardized residuals and residuals squared indicate some misspecification of the model.

The evidence, in short, is that the EARCH-in-mean model considered here reveals a mild correlation between the term premium and conditional variance. Furthermore, this model provides no convincing evidence for important nonlinearities which leads the rejection of the expectations hypothesis.

However, the EGARCH-in-mean model considered so far is relatively restrictive and may be subject to misspecifications which bias the estimation result strongly. First, our EGARCH variance equation may be misspecified, which leads to biased and inconsistent estimates of the mean equation (PAGAN and ULLAH, 1988). Second, our mean equation may be misspecified: The term premium is a deterministic function of the short rate volatility and contains no other element. The relevance of these problems is investigated by utilizing the bivariate seminonparametric nonlinear impulse response analysis developed by GALLANT, ROSSI and TAUCHEN (1993). To motivate the use of this approach consider the following general spread equation obtained in the framework of the expectations theory,

$$R_t - r_t = \theta\left(\sigma^2{}_{t+1}\,(\varepsilon_t, \varepsilon_{t-1}, \ldots) \right) + \frac{1}{3}\sum_{j=1}^{4}\left(2\Delta r^e_{t+j} + \Delta r^e_{t+j+4} \right) + \eta_t\,. \tag{1a}$$

The term premium depends not only on expected short rate changes and an idiosyncratic shock η_t, but also on a time varying term premium depending on the conditional variance of the short rate σ^2_{t+1}, which is in turn influenced by a constant and lagged short rate shocks. Equation (1a) shows that a short rate shock ε_t affects the movement in the spread via two channels. First, it has an impact on expected future short rate changes and therefore has an impact on the spread movement. Second, it influences the term premium by its effect on short rate volatility. The method of GALLANT, ROSSI and TAUCHEN allows us to estimate the dynamic effect of a short rate shock on the spread, on short rate changes and on the volatility of the short rate without explicitly specifying the term premium-

(θ) and the volatility- (σ^2_{t+1}) functions. For example, assume that we find no effect of a positive short rate shock on future short rate changes, whereas a strong effect on short rate volatility is revealed. Given equation (1a) this implies that, if the term premium is volatility dependent, a positive as well as a negative short rate shock would have a positive dynamic influence on the spread.

3. SNP ESTIMATION AND NONLINEAR IMPULSE RESPONSE

In this section, we first apply the SNP estimator suggested by GALLANT and NYCHKA (1987) and GALLANT and TAUCHEN (1989, 1992) to estimate directly the bivariate one-step ahead conditional density of short rate changes and the spread. After the joint density is estimated, the nonlinear impulse response technique developed by GALLANT, ROSSI and TAUCHEN (1993) is applied to explore the mean and volatility dynamics of these two variables without relying on specific functional forms.

3.1 SNP Estimation of the Conditional Density

The basic idea of the SNP estimator is to approximate a density function by a truncated Hermite expansion of the form

$$h(z) \propto [p(z)]^2 \phi(z) \tag{4}$$

where z denotes a M-dimensional vector, $p(z)$ is a multivariate polynomial of degree K_z $\phi(z)$ and denotes the density of a two dimensional Gaussian process. One can obtain a more complex specification by means of a change of variables

$$y = Rz + \mu\,, \tag{5}$$

where R is an upper triangular matrix and μ is a vector of dimension M. Consequently, we obtain the following expression:

$$f(y|\theta) \propto \left\{ p\left[R^{-1}(y-\mu) \right] \right\}^2 \left\{ \phi\left[R^{-1}(y-\mu) \right] / |\det(R)| \right\}. \tag{6}$$

Thus, the leading term of the expansion is a multivariate Gaussian density with mean and variance-covariance matrix $\Sigma = RR'$. When K_z is set to be zero, one gets a Gaussian density. When K_z is positive, one gets a density that can accommodate fat tails and

Table 2: SNP

L_u	L_r	L_p	K_Z	I_z	K_x	I_x	p_θ	Obj	Schwarz	Hannan–Quinn
0	0	0	0	0	0	0	5	2.79694	2.81878	2.80940
1	1	0	0	0	0	0	11	1.97159	2.01964	1.99901
2	2	0	0	0	0	0	17	1.90690	1.98169	1.94927
3	3	0	0	0	0	0	23	1.86384	1.96432	1.92117
4	4	0	0	0	0	0	29	1.84979	1.97648	1.92208
5	5	0	0	0	0	0	35	1.83333	1.98623	1.92057
6	6	0	0	0	0	0	41	1.82530	2.00442	1.92750
5	6	0	0	0	0	0	37	1.82876	1.99040	1.92099
5	7	0	0	0	0	0	39	1.82856	1.99894	1.92577
4	5	0	0	0	0	0	31	1.83424	1.96967	1.91151
4	6	0	0	0	0	0	33	1.82980	1.97397	1.91205
4	5	0	1	0	0	0	33	1.83249	1.97666	1.91475
4	5	0	2	1	0	0	35	1.83242	1.98533	1.91967
4	5	0	2	0	0	0	36	1.81737	1.97465	1.90711
4	5	0	3	2	0	0	37	1.82374	1.98539	1.91597
4	5	0	3	1	0	0	38	1.81522	1.98123	1.90994
4	5	0	3	0	0	0	40	1.79931	1.97406	1.89901
4	5	0	4	3	0	0	39	1.72090	1.89128	1.81811
4	5	0	4	2	0	0	40	1.71303	1.88778	1.81274
4	5	0	4	1	0	0	42	1.70299	1.88648	1.80768
4	5	0	4	0	0	0	45	1.67794	1.87454	1.79011
4	5	0	5	4	0	0	41	1.72087	1.89999	1.82306
4	5	0	5	3	0	0	42	1.71275	1.89624	1.81745
4	5	0	5	2	0	0	44	1.70655	1.89878	1.81623
4	5	0	5	1	0	0	47	1.67698	1.88231	1.79413
4	5	0	5	0	0	0	51	1.67408	1.89689	1.80121
4	6	0	4	0	0	0	47	1.67704	1.88237	1.79419
4	7	0	4	0	0	0	49	1.67574	1.88981	1.79787
4	8	0	4	0	0	0	51	1.67334	1.89615	1.80047
4	5	1	4	3	1	0	57	1.65584	1.90486	1.79792
4	5	1	4	2	1	0	60	1.64674	1.90887	1.79630
4	5	1	4	1	1	0	66	1.61805	1.90640	1.78257
4	5	1	4	0	2	1	105	1.54173	2.00046	1.80346
4	5	1	4	0	1	0	75	1.59870	1.92636	1.78565
4	5	1	4	0	2	0	120	1.51865	2.04292	1.81777

L_u	L_r	L_p	K_Z	I_z	K_x	I_x	p_θ	**Obj**	**Schwarz**	**Hannan–Quinn**
4	5	1	5	2	1	0	72	1.61263	1.92718	1.79210
4	5	1	5	0	1	0	93	1.59001	1.99631	1.82182
4	5	1	5	2	2	1	100	1.55114	1.98802	1.80040
4	5	1	5	2	2	0	114	1.52843	2.02648	1.81259
4	5	1	5	0	2	0	156	1.48503	2.16657	1.87388
4	6	1	4	0	1	0	77	1.59750	1.93390	1.78943
4	6	1	4	0	2	1	107	1.54089	2.00836	1.80760
4	6	1	5	2	2	1	102	1.55009	1.99572	1.80434
4	6	1	5	2	2	0	116	1.52736	2.03415	1.81651
4	6	1	5	1	2	1	117	1.52630	2.03745	1.81794
4	7	1	4	0	1	0	79	1.59525	1.94039	1.79217
4	7	0	5	2	0	0	48	1.70279	1.91249	1.82244
4	7	1	5	2	2	1	104	1.55001	2.00437	1.80924
4	7	1	5	2	2	0	118	1.52725	2.04278	1.82138
4	8	1	4	0	1	0	81	1.59305	1.94693	1.79496
5	6	1	5	2	2	1	106	1.54196	2.00506	1.80618
5	6	1	5	2	2	0	120	1.52032	2.04458	1.81944
5	7	1	5	2	2	0	122	1.52027	2.05327	1.82437

Table 2: (Contin.)

Specification tests *														
							Mean				Variance			
							γ_1		γ_2		γ_1		γ_2	
L_u	L_r	L_p	K_z	I_z	K_x	I_x	F	p-value	F	p-value	F	p-value	F	p-value
4	5	1	4	0	0	0	2.5274	0.0005	5.4640	0.0000	7.1342	0.0000	9.7984	0.0000
4	5	1	4	1	1	0	2.1314	0.0041	3.5998	0.0000	3.7074	0.0000	3.9449	0.0000
4	5	1	4	0	1	0	0.7158	0.7986	2.1695	0.0034	1.5458	0.0683	2.9702	0.0001
4	5	1	4	0	2	0	1.7238	0.0310	1.8851	0.0144	1.9819	0.0089	1.9629	0.0098
4	5	1	5	2	1	0	2.2422	0.0023	3.6223	0.0000	3.8090	0.0000	3.6241	0.0000
4	5	1	5	2	2	1	1.8183	0.0200	1.9332	0.0113	1.5910	0.0563	2.4647	0.0007
4	6	1	5	2	2	1	1.7799	0.0239	1.9101	0.0127	1.6071	0.0537	2.3936	0.0010
4	6	1	5	2	2	0	1.7191	0.0317	1.8514	0.0170	1.6277	0.0479	2.3209	0.0015
4	7	1	5	2	2	0	1.7948	0.0222	1.9334	0.0113	1.5972	0.0548	2.4522	0.0007
5	6	1	5	2	2	1	1.9071	0.0129	1.9487	0.0105	1.8833	0.0145	2.3336	0.0013
5	6	1	5	2	2	0	1.7525	0.0271	1.9656	0.0097	1.9116	0.0281	2.2855	0.0018
5	7	1	5	2	2	0	1.7602	0.0262	1.977	0.0091	1.7556	0.0268	2.3117	0.0016

* For the mean, the specification test is a regression of the standardized residuals on three lags of a linear, quadratic, and cubic term in each variable; for the variance, it is from the same regression except the dependent variable is the squared standardized residual.

skewness. Other modification can be obtained by letting μ and R be a linear function of past values of y to accommodate a VAR and ARCH specification. The number of lags in μ and R are denoted as L_u and L_r, respectively. To approximate a conditionally heterogeneous process, each coefficient of the polynomial $p(z)$ can be modeled as a polynomial of degree K_z in the past values of y. The new polynomial $p(z)$, where x is the vector of lagged values of y, is hence of degree $K_z + K_x$. The number of lags in $p(z, x)$ is denoted as L_p.

To determine the best fit, we use the Schwarz and Hannan-Quinn model selection criteria together with specification tests on the conditional mean and variance. The specification tests on conditional mean is a regression of each of the standardized residuals ($[\mathrm{Var}_{t-1}(y_t)]^{-1/2}[y_t - E_{t-1}(y_t)]$) on a constant and lagged y up to the third power, i.e. $\{y_{t-k}, y_{t-k} \otimes y_{t-k}, y_{t-k} \otimes y_{t-k} \otimes y_{t-k}\}_{k=1}^{3}$. The specification test on the conditional variance is from the same regression, except the dependent variable is the squared standardized residual.

The estimation results are presented in Table 2. The two additional parameters I_z and I_x are the degrees of suppression in the interactions of the variable in the polynomial . From Table 2, the Schwarz and Hannan-Quinn preferred model are $L_u = 4$, $L_r = 5$, $L_p = 0$, $K_z = 4$, $K_x = 0$ and $L_u = 4$, $L_r = 5$, $L_p = 1$, $K_z = 4$, $I_z = 1$, $K_x = 1$, respectively. However, both models fail the diagnostics for both the conditional mean and variance of the specification tests drastically, which indicates some predictability in the standardized residuals and their squares. This suggests that an increase of the size of the model is necessary. Further expansion of to $L_u = 4$, $L_r = 5$, $L_p = 1$, $K_z = 4$, $I_z = 0$, $K_x = 2$, $I_x = 0$ provides a final model that passes all the specification tests at almost one-percent level. The final model incorporates additional conditional heterogeneity beyond ARCH via $L_p = 1$, $K_x = 2$. With the number of parameters $p_\theta = 120$, the estimated one-step ahead conditional density of Swiss spread and short rate changes is essentially highly non-Gaussian and nonlinear. It is an ARCH model with a non-normal error density. And we use this model for the subsequent impulse response analysis.

3.2 *Impulse Response Analysis*

We now turn to the impulse response analysis to investigate the empirical relevance of a volatility dependent term premium in our SNP framework. To this end we employ the method developed by GALLANT, ROSSI and TAUCHEN (1993) which consists of computing response profiles of the conditional mean and conditional variance. According to GALLANT et al, the conditional mean profile given initial condition x^0 is

$$\hat{y}_j(x^0) = E(y_{t+j} | x_t = x^0) = \hat{y}_j^0 ,$$

for $j = 1, ..., J$. If x^0 is changed by

$$x^+ = x^0 + \delta$$

or

$$x^- = x^0 - \delta$$

for some realistic value δ in the arguments of the conditional density, the J-step conditional mean profile becomes

$$\hat{y}_j(x^+) = E(y_{t+j}|x_t = x^+) = \hat{y}_j^+$$

for $x^+ = x + \delta$ and

$$\hat{y}_j(x^-) = E(y_{t+j}|x_t = x^-) = \hat{y}_j^-$$

for $x^- = x - \delta, j = 1$. Accordingly, the positive and negative impulse response of J-step conditional mean are

$$\{ \hat{y}_j(x^+) - \hat{y}_j(x^0) \}_{j=1}^{J}$$

and

$$\{ \hat{y}_j(x^-) - \hat{y}_j(x^0) \}_{j=1}^{J},$$

respectively. These two terms provide a natural measurement to study the effect of the «shock» δ on the conditional mean of the system.

Analogous to the conditional mean, one can measure the effects of perturbing conditional arguments on the J-step ahead conditional variance. Since

$$\begin{aligned}\hat{V}_j(x^0) &= \mathrm{Var}(y_{t+j}|x_t = x^0) \\ &= E\left\{ \left[y_{t+j} - E\,(y_{t+j}|x_t = x_0) \right] \times \left[y_{t+j} - E\,(y_{t+j}|x_t = x_0 \right] |x_t = x_0 \right\}\end{aligned}$$

for $j = 1, \ldots, J$. Similarly,

$$\hat{V}_j(x^+) = \mathrm{Var}(y_{t+j}|x_t = x^+)$$

and

$$\hat{V}_j(x^-) = \mathrm{Var}(y_{t+j}|x_t = x^-),$$

for $j = 1, \ldots, J$, we get the positive and negative impulse responses of perturbations δ on the volatility which are

$$\{ \hat{V}_j(x^+) - \hat{V}_j(x^0) \}_{j=1}^{J}$$

and

$$\{ \hat{V}_j(x^-) - \hat{V}_j(x^0) \}_{j=1}^{J}$$

respectively.

The conditional mean and variance profiles are computed for both short rate changes and long-short spread. We investigate the effects of four types of shocks which are designed by inspection of the data scatter plot to generate different combinations of some typical and realistic perturbations:

A shock:

$$\delta y_1^{A+} = 1.00\sigma_{\Delta r} \qquad \delta y_2^{A+} = 0.00$$
$$\delta y_1^{A-} = -1.00\sigma_{\Delta r} \qquad \delta y_2^{A-} = 0.00$$

B shock:

$$\delta y_1^{B+} = 0.00 \qquad \delta y_2^{B+} = 1.00\sigma_S$$
$$\delta y_1^{B-} = 0.00 \qquad \delta y_2^{B-} = -1.00\sigma_S$$

C shock:

$$\delta y_1^{C+} = 1.00\sigma_{\Delta r} \qquad \delta y_2^{C+} = 1.00\sigma_S$$
$$\delta y_1^{C-} = 1.00\sigma_{\Delta r} \qquad \delta y_2^{C-} = -1.00\sigma_S$$

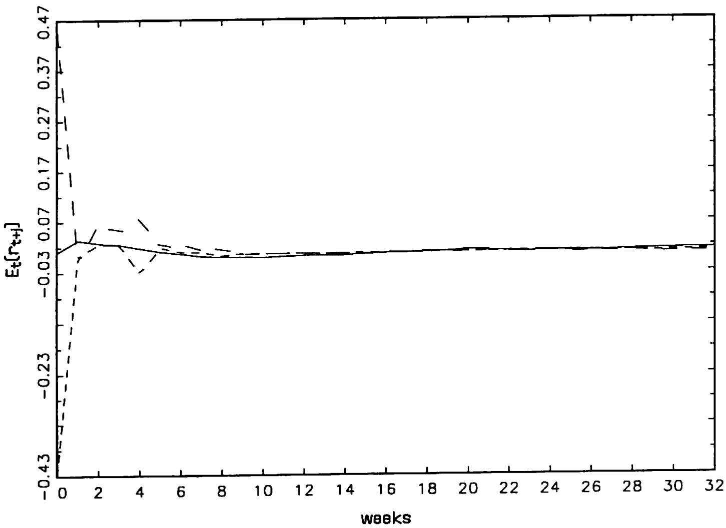
D shock:

$$\delta y_1^{D+} = -1.00\sigma_{\Delta r} \qquad \delta y_2^{D+} = 1.00\sigma_S$$
$$\delta y_1^{D-} = -1.00\sigma_{\Delta r} \qquad \delta y_2^{D-} = -1.00\sigma_S$$

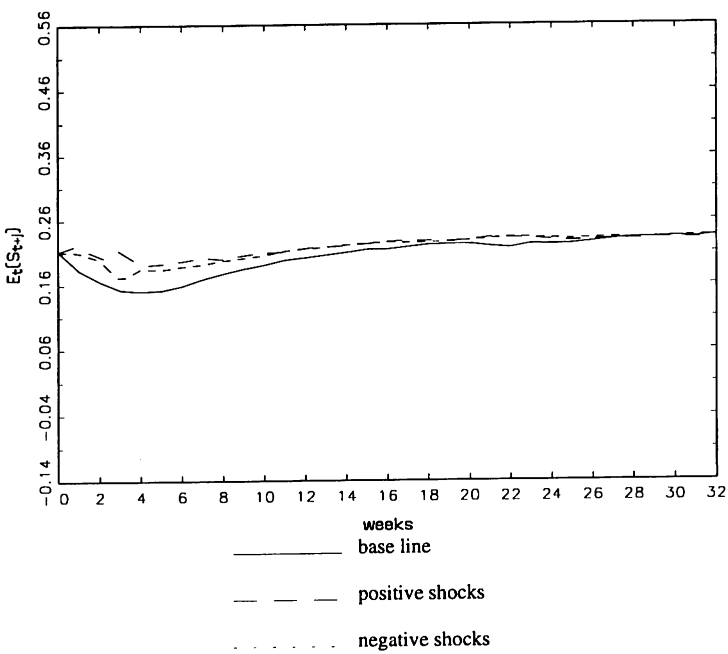
where $y_1 = \Delta r_t$, $y_2 = R_t - r_t$, and $\sigma_{\Delta r}$ and σ_S are the sample standard deviation of the short rate changes and the spread, respectively.
In this design, *A* shock reflects a pure short rate movement up or down by one standard deviation, whereas *B* shock reflects a pure spread movement up or down by one standard error. *C* shock combines a spread movement together with a positive short rate shock, whereas *D* shock combines a spread movement with a negative short rate shock. In our analysis we are primarily interested in the results for the pure short rate shocks. However, the other three shocks are of interest for the following reasons. First, since the expectations theory also implies that the spread reflects future short rate changes, we would expect some effects of spread shocks (*B* shock) on the subsequent short rate dynamics. Second, being aware of the potential relevance of the contemporaneous correlation structure between the two variables in a nonlinear system [pointed out by GALLANT, ROSSI and TAUCHEN (1993)], the *C* and *D* shocks are included.

Figure 1: Impulse Response of Short Rate and Spread to A Shocks

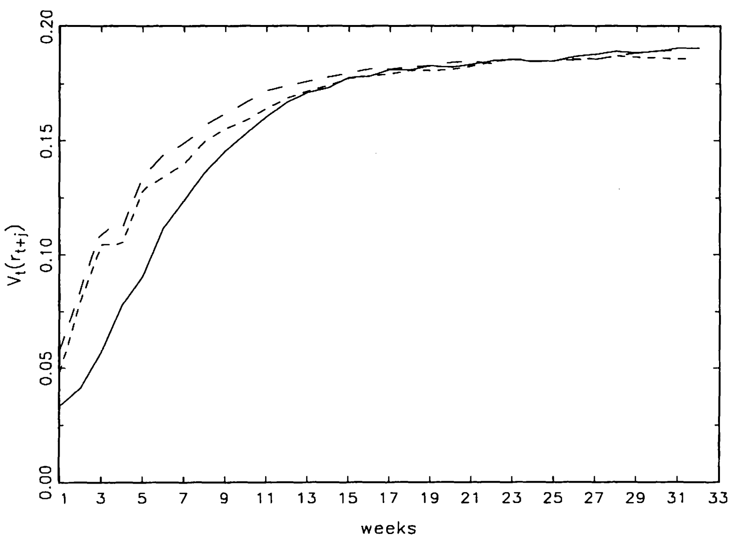
(a) Short



(b) Spread



(c) Volatility of Short



(d) Volatility of Spread

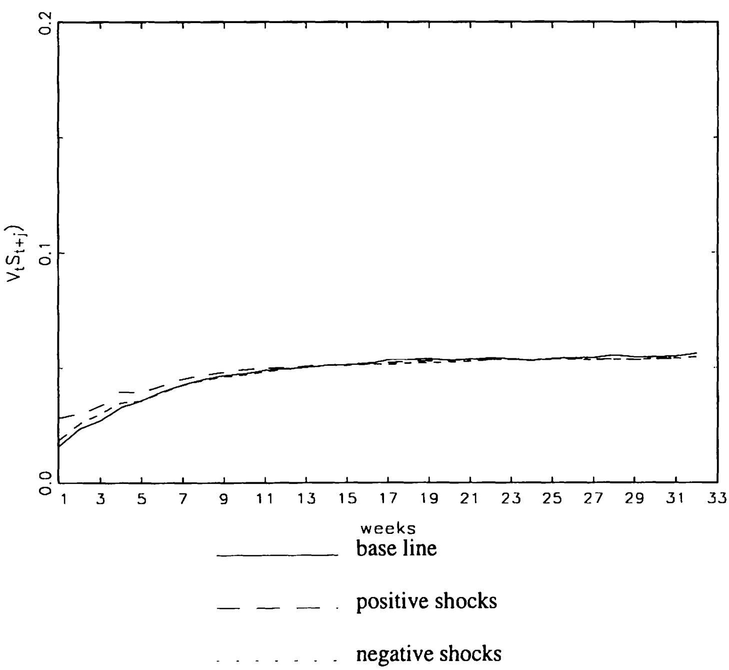
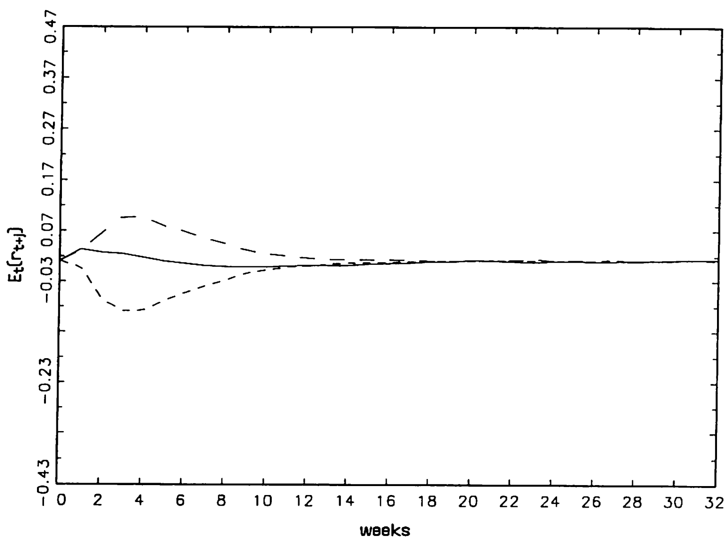
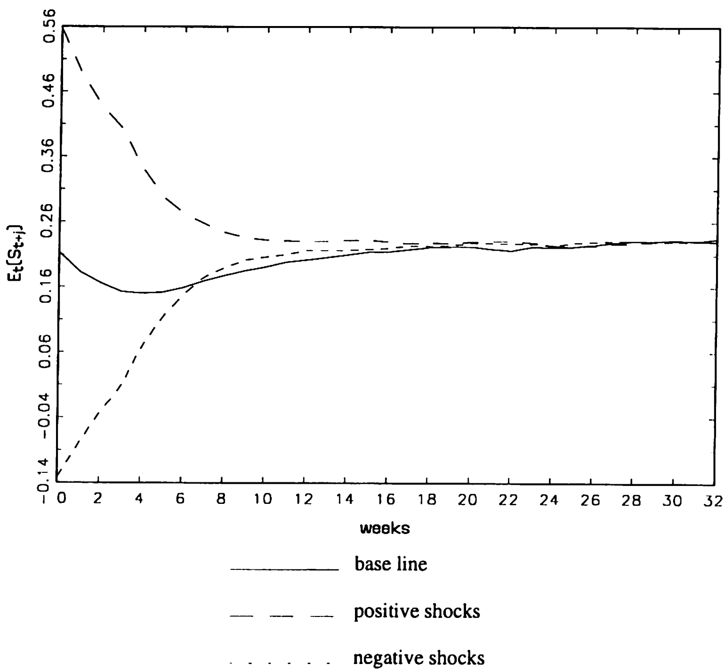


Figure 2: Impulse Response of Short Rate and Spread to B Shocks

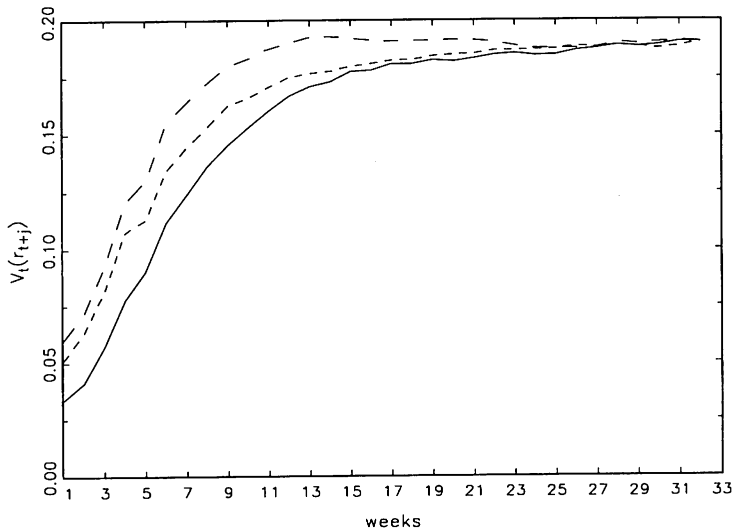
(a) Short



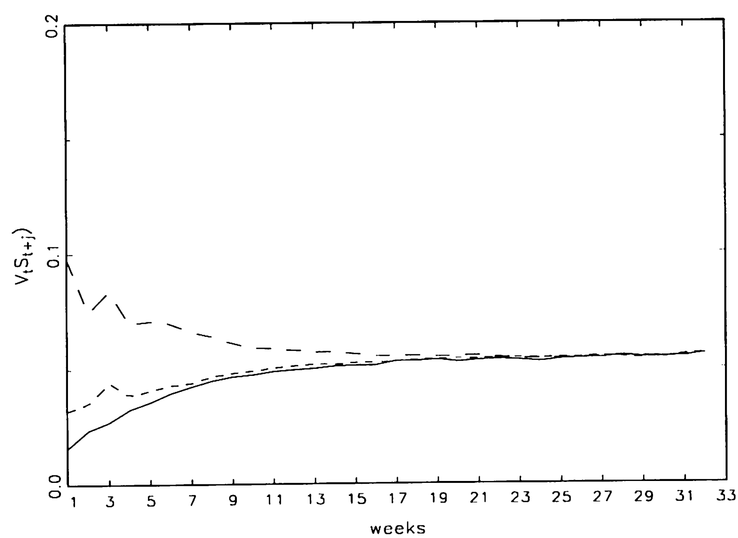
(b) Spread



(c) Volatility of Short



(d) Volatility of Spread



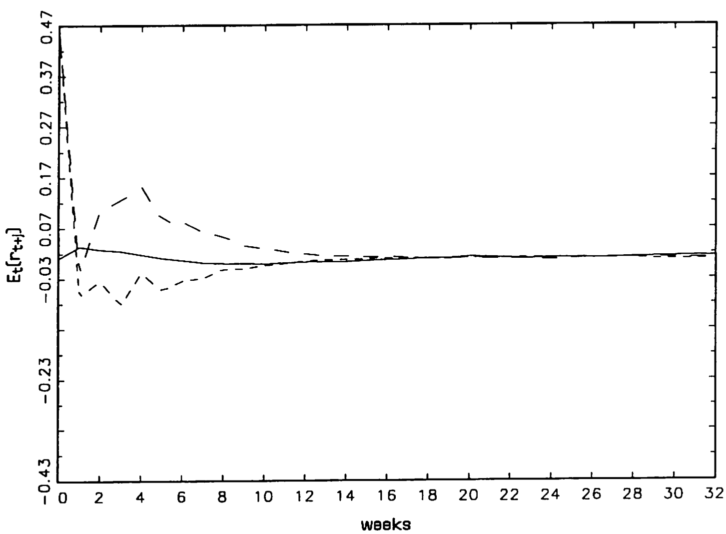
base line

positive shocks

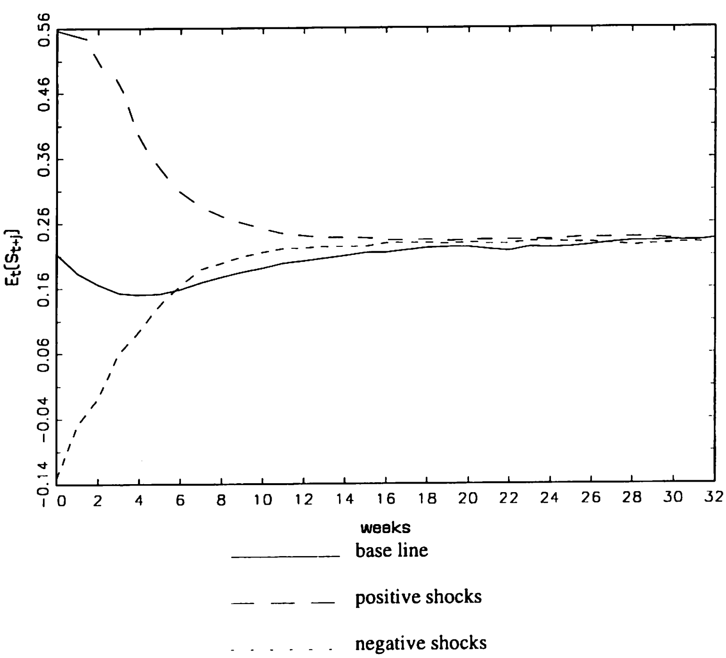
negative shocks

Figure 3: Swiss Interest Rates: Impulse Response of Short Rate and Spread to *C* Shocks

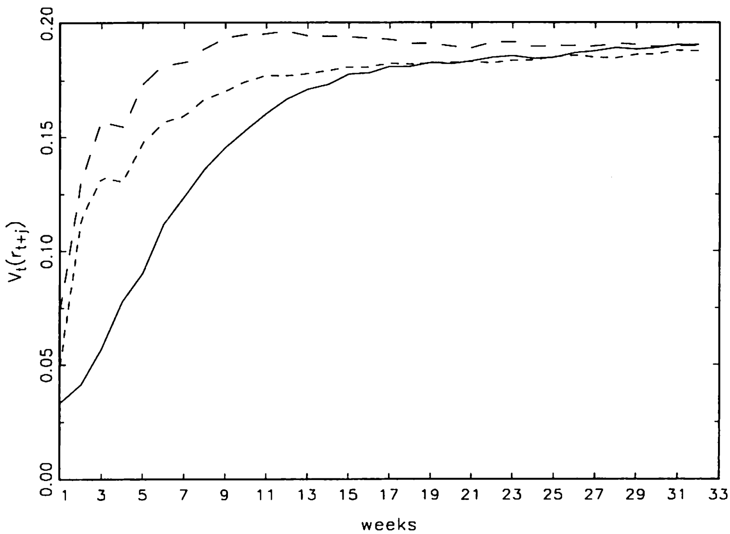
(a) Short



(b) Spread



(c) Volatility of Short



(d) Volatility of Spread

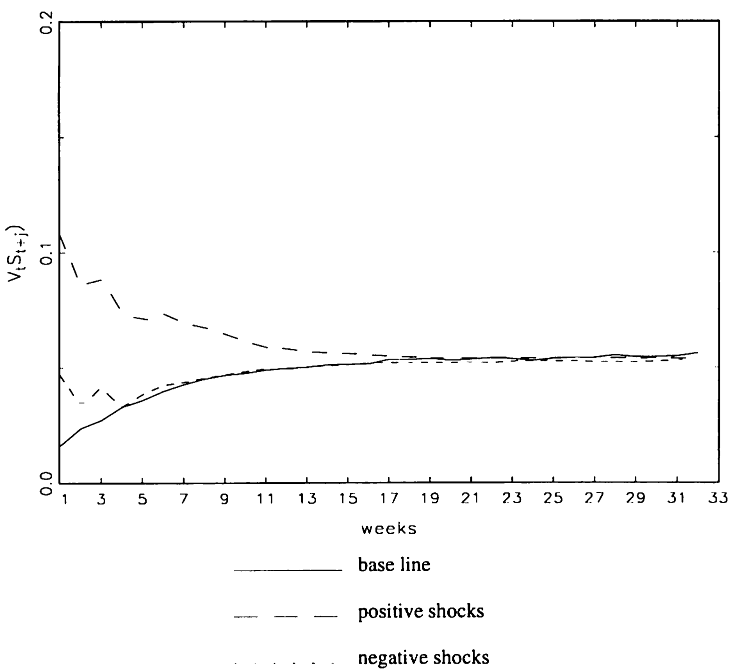
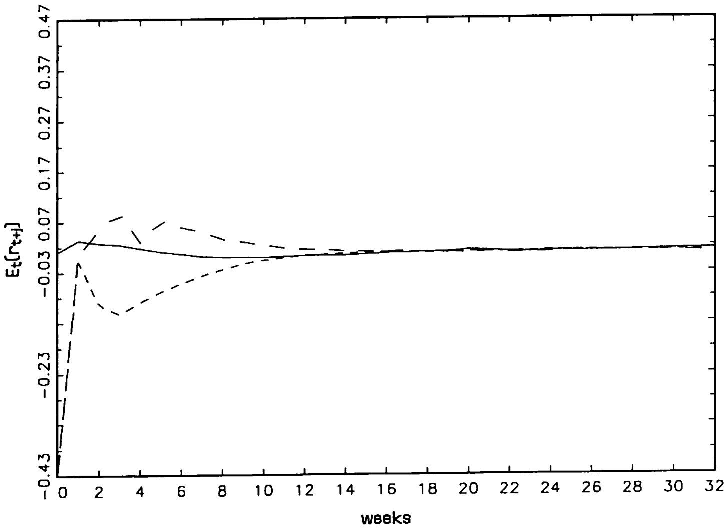
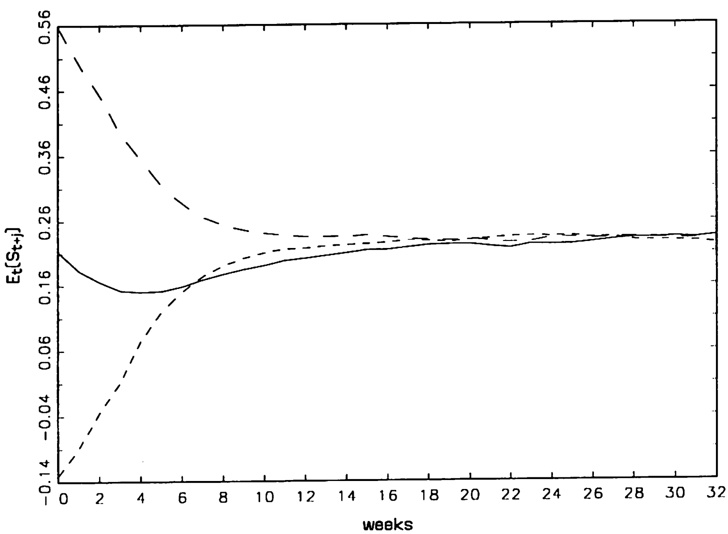


Figure 4: Swiss Interest Rates: Impulse Response of Short Rate and Spread to *D* Shocks

(a) Short



(b) Spread

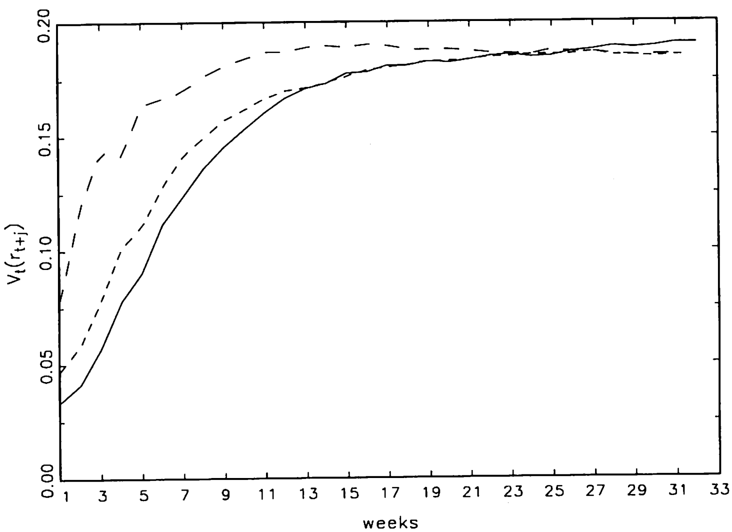


———— base line

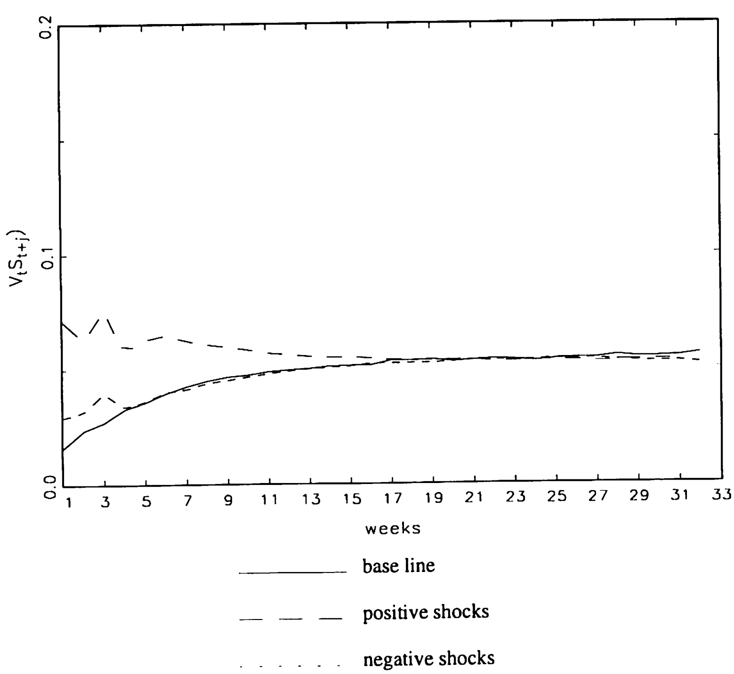
— — — positive shocks

. . . . . . negative shocks

(c) Volatility of Short



(d) Volatility of Spread



The impulse response profiles for the Swiss three and one month interest rate are summarized in Figure 1 to Figure 4. Figure 1 shows the impulse responses of the two series to the short rate shocks (type ***A***). The mean responses as well as the volatility responses of the short rate are symmetric, which confirm the estimation results of the EGARCH model estimates reported in Table 1 in which no asymmetric effect is found. However, the adjustment pattern is different from that implied by the EGARCH(1,1) model: shocks to variance is not highly persistent as one would expect from the estimation result from the EGARCH model which indicates a highly persistent volatility process ($\hat{b} = 0.972$). In addition, we find only a very weak (near zero) effect of the short rate shocks on future short rate changes. However, the effect on volatility is rather strong. Weak mean responses but strong volatility responses imply that we should find a strong positive effect on the conditional mean if a volatility dependent term premium exists. Figure 1(b) shows only a weak and insignificant reaction of the spread to the short shock. Thus, our nonlinear impulse response results confirm the insignificant conditional mean volatility relationship found in the EGARCH modeling framework.

Figure 2 contains the results of the pure spread shocks (type ***B***). The mean responses of the spread displayed in Figure 2(b) is slowly damped and rather symmetric about the baseline. Fig. 2(d) shows an asymmetric volatility response of the spread to its own shocks: the impact of a positive spread shock is much stronger and last longer than a negative shock.

Figure 2(a) and 2(c) show impulse responses of short rate to the type ***B*** shocks. These moderate spread movements ($\sigma_S = 0.34413$) have significant impact on the subsequent short rates dynamics: the effects of the spread shocks on the short rate volatility are slowly damped, whereas the effects on the conditional mean of the short rate last shorter.

Figure 1(b), (d) and Figure 2(a), (c) show that spread shocks have significant impact on the subsequent short rates dynamics, whereas short rate shocks have neither effect on subsequent spread mean nor on volatility. The mild feedback from short rate to spread indicates that the long-short spread Granger causes the short rate. This result confirms the expectations theory at a qualitative level.

For the combined ***C*** and ***D*** shocks in Figures 3 and 4, by comparing the previous figures, it shows that the volatility responses are dominated by their own shocks. The mean responses are dominated by the spread shocks, the effects which can be obtained by adding the corresponding ***A*** and ***B*** type shocks. This suggests that the system we analyze behaves almost like a linear one with this respect: The joint occurrence of the shocks does not seem to influence their impact.

4. CONCLUSION

In this paper we analyze the relationship of the term premium and the volatility of the short rate empirically using weekly data for one and three months Swiss interest rate data. First, we apply an EGARCH-in-mean framework to test the expectations hypothesis with a volatility dependent term premium. The corresponding estimates point to no significant influence of volatility on the term premium and reject the expectations hypothesis. Second, we apply the seminonparametric nonlinear impulse response analysis proposed by GALLANT, ROSSI and TAUCHEN (1993) to the short rate change and the long short spread. This framework allows us to analyze the term premium volatility relationship without using a specific, parametric nonlinear model such as EGARCH-in-mean. The latter analysis confirms the EGARCH results, although the volatility dynamics estimated are clearly different from that implied by the EGARCH estimate. In addition, the impulse response analysis exhibits a symmetric and additive reaction of the conditional mean to various shocks. For the conditional variance a clearly milder reaction to negative spread shocks than to positive spread shocks is found. Moreover, the spread has a dynamic influence on the short rate whereas the converse is not true. This result confirms a qualitative implication of the expectations hypothesis. Our conclusion is that the nonlinear mean interest rate dynamics are not of great importance to account for the rejection of the expectations hypothesis for Swiss data. It seems more promising to analyze linear time variant term premium models such as the policy reaction framework introduced by MCCALLUM (1994) and generalized by KUGLER (1995), which has been shown to work well for Swiss and other data.

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ZUSAMMENFASSUNG

In dieser Arbeit wird die Beziehung zwischen der Terminprämie und der Volatilität der kurzfristigen Zinssätze mit Schweizerischen Wochendaten für die Jahre 1978-1992 untersucht. Sowohl ein parametrisches EGARCH-in-mean Einzelgleichungsmodell als auch eine seminichtparametrische nichtlineare Impulse Response Analyse ergeben keine klare empirische Evidenz für eine Terminprämie-Volatilitätsbeziehung. Dieses Ergebnis deutet darauf hin, dass die nichtlineare Dynamik des bedingten Mittels keine grosse Bedeutung für die Interpretation der negativen Resultate von Tests der Erwartungshypothese der Zinsstruktur hat.

SUMMARY

In this paper we investigate the relationship between the term premium and the volatility of the short interest rate by applying a single equation EGARCH-in-mean model as well as bivariate seminonparametric nonlinear impulse response analysis to weekly Swiss data over the period from 1978 to 1992. The estimation results of the restrictive parametric as well as the general seminonparametric methods provide no convincing evidence for a term premium volatility relationship. Our finding indicates that the nonlinear conditional mean dynamics plays a small role in accounting for the rejection of the expectations hypothesis of the term structure of interest rates.

RESUME

Cet article examine la relation entre la prime à terme et la volatilité des taux d'intérêt avec des dates hebdomadaires suisses pour les années 1978-1992. Un modèle paramétrique à équation unique EGARCH-in-mean ainsi qu'une analyse impulse response

seminonparamétrique et nonlinéaire sont appliqués aux dates. Les résultats ne permettent pas de conclusion définitive concernant une relation entre la prime à terme et la volatilité des taux d'intérêt. Ce résultat indique que la dynamique nonlinéaire de la moyenne conditionelle ne joue pas un rôle important pour l'interprétation des résultats rejetant l'hypothèse expectative de la structure des taux d'intérêt.