Pollution Control in a Stochastic Environment

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1. INTRODUCTION

When should the government adopt a particular policy (e.g., to control pollution) in response to a perceived threat to the environment? The standard framework used in economics to answer this question is cost-benefit analysis. Consider, for example, that the government imposes a carbon tax to reduce global warming. By distorting relative prices, this policy would impose an expected flow of costs on society in excess of the government tax revenues it generates. Presumably, it also yields an expected flow of net benefits to society. Households and firms would burn less fuel, less carbon dioxide would accumulate in the atmosphere, global mean temperatures would be correspondingly lower. The standard framework would recommend this policy if the present value of the expected flow of benefits exceeds the present value of the expected flow of costs.

This standard approach, however, ignores at least three important characteristics of most environmental problems and the policies designed to respond to them: (1) There is considerable uncertainty over the future costs and benefits of adopting a particular policy. With global warming, for example, there is uncertainty over how much average temperatures are likely to rise with or without reduced CO_2 emissions, and there is also uncertainty over the economic impact of higher temperatures. (2) There are usually important irreversibilities associated with environmental policy. Irreversibilities may arise with respect to environmental damage itself but also with respect to investments in abatement capital. (3) The adoption of a particular environmental policy is rarely a now-or-never proposition. In most cases, the government can delay action, wait for new information, and choose intermediate levels of control. Hence, the question of the optimal timing of an environmental policy arises.

This paper deals with the fact that environmental decision making is characterized by both uncertainty and irreversibility. We shall assume that uncertainty arises because future net benefits of pollution control are not known with certainty, probably due to a lack of knowledge about the timing and nature of ecosystem recovery, and the willingness of individuals to pay for the goods and services it can produce. In this paper, the

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This paper was written while the first author was a visiting Research Fellow at the University of California at Berkeley, USA. The first author also gratefully acknowledges financial support by the Swiss National Science Foundation. We would like to thank an anonymous referee for helpful comments.

irreversibility is associated with the assumption that investments in pollution are so specialized that, once made, capital can never be retrieved for use in another activity.

It goes without saying that irreversibilities associated with environmental damage itself is also a very important problem. In this paper, we capture this feature in the special way that there are irreversible changes after a certain point in the future. It is assumed that the occurence of the critical period cannot be influenced by the decision maker.

To this end, we shall allow for the possibility of discontinuities in the evolution of net benefits of control. Specifically, net benefits of control may take a sudden jump to zero at some point in time due to the existence of a «critical period» after which it never pays to control pollution anymore. The existence of a critical period may be the result of threshold levels that trigger irreversible changes after continued stress on an ecosystem.

To capture uncertainty and irreversibility effects, we shall use stochastic dynamic programming. In general, environmental and resource economics would appear to be a fertile field for the application of stochastic dynamic programming. This theory has originally been developed in finance and has then been applied to a wide range of other economic fields like problems of opening and closing a mine (BRENNAN and SCHWARTZ 1985), making a specialized investment (MCDONALD and SIEGEL 1986), developing land (CLARKE and REED 1990), or preserving old-growth forest (REED 1993, CONRAD 1994). In a very recent book, DIXIT and PINDYCK (1994) provide a comprehensive overview of applications of stochastic dynamic programming to a variety of economic problems.

Of course, uncertainty and irreversibility have been analyzed in the literature on environmental economics before, most notably beginning with the seminal papers of ARROW and FISHER (1974) and HENRY (1974). The main focus, however, has been on the economics of land development and wilderness protection. More closely related to our framework is a paper by FISHER and HANEMANN (1989) who analyze environmental decision making in a 3-period framework. The procedure here, however, departs in several important aspects from their approach: (1) We consider a more general continuous-time infinite horizon setting, (2) we allow for intermediate levels of control rather than «all-or-nothing» decisions resulting from linear benefit functions, and (3) the evolution of social valuation of pollution control benefits over time is modelled by using a stochastic process.

The main results of this paper may be summarized as follows: (1) Optimal pollution control involves a sequence of decisions through time, reflecting the changing social valuation of pollution control measures. And, increasing uncertainty of the social valuation of control benefits leads to a lower degree of control if investments in control facilities are irreversible; (2) using a certainty equivalence approach where random variables are replaced by their expected values leads to a higher degree of pollution control than it is optimal, reflecting the fact that this approach does not take into account the value of future information; and (3) the existence of a «critical period» leads to a *lower* optimal degree of pollution control compared to a world without a critical period. The last result is due to the fact that, in our model, the policy maker cannot influence the occurrence of such a period, therefore future benefits of control are discounted more heavily.

The paper is organized as follows: Section 2 sets up the basic stochastic model and derives an optimal pollution control policy; section 3 compares the optimal policy with the certainty equivalence approach. Section 4 focuses on the effect of a critical period on the optimal policy. Finally, section 5 offers some concluding remarks.

2. OPTIMAL POLLUTION CONTROL POLICY

We consider an economy where an environmental authority chooses whether or not to control pollution, and is concerned with the net present value (benefits minus costs) of pollution control. The decision to control is embodied as investment in abatement capital. Obviously, the optimal control is defined as the choice that maximizes this value. The environmental agency faces the following constraints: (1) Investments in abatement capital are irreversible. Once one invests in pollution control facilities, it will be useful only for pollution control and cannot later be «un-invested». (2) The benefits of control are uncertain, for example, due to a lack of knowledge about the timing and nature of ecosystem recovery and the willingness of individuals to pay for control facilities and their services. (3) Later on, we shall suppose that beyond some point in the future, failure to control is irreversible due to the existence of a «critical period» after which benefits of control drop to zero and remain there forever.

Let the outcome of the decision made by the environmental agency at instant t be denoted by $X(t)$. We can think of $X(t)$ being the current stock of «abatement capital». A higher stock of abatement capital corresponds to a higher level of pollution control. We assume that pollution control investment takes place in a continuous setting. That is, we allow for the case that the environmental authority chooses an intermediate level of control: the rate of investment in control facilities is denoted by $u(t) \in [0,1]$ where $u(t) = 0$ may be construed as «no control» and $u(t) = 1$ as «maximal control» at instant t, respectively. *I.e.*, investment in abatement capital is bounded from below by zero and bounded from above by one, perhaps due to capacity constraints. Associated with the current level of pollution control is a flow of benefits and costs. Suppose that the costs associated with investing in abatement capital at a rate $u(t)$ are given by $cu(t)$ where $c > 0$, a constant. The benefits resulting from pollution control are represented by a twice differentiable, increasing, and concave function $Q(X(t))$ with $Q'(X(t)) > 0$ and $Q''(X(t)) < 0$.

Furthermore, suppose that future benefits of pollution control are uncertain, and that the instantaneous flow of net benefits can be described by the welfare function

$$W(t, X(t)) = v(t)Q(X(t)) - cu(t) \tag{1}$$

where $v(t)$ is the «social valuation» placed on the services yielded by control facilities at instant t. Q(X(t)) then measures the size of the service flows from the current stock of abatement capital. Further, suppose that $v(t)$ is a stochastic process which evolves through time as geometric Brownian motion with «drift» governed by the stochastic differential equation

$$\frac{dv}{v} = \alpha dt + \sigma dz \tag{2}$$

where α is a drift parameter representing constant upward or downward trends in the valuation of control benefits, and σ^2 is the instantaneous variance. Moreover, $z(t)$ is a standard Wiener process, so that dz is «white noise». Note that the acquisition of information about v over time does not depend on the control decision itself; it simply emerges with the passage of time. Thus, in our framework, learning is unrelated to prior control decisions, in contrast, for example, to the Bayesian learning model of MILLER and LAD (1984). Defining $q = \ln v$, the stochastic differential of v is

$$dq = mdt + \sigma dz \tag{3}$$

where $m = \alpha - (\sigma^2/2)$. This expression can be derived by using Ito's Lemma (see, e.g. PINDYCK 1991): From (2), we get

$$\begin{aligned} dq &= \frac{1}{v}dv - \frac{1}{2}v^2(dv)^2 \\ &= \alpha dt + \sigma dz - \frac{1}{2}\sigma^2 dt \\ &= (\alpha - \frac{1}{2}\sigma^2)dt + \sigma dz\,. \end{aligned}$$

With $v = e^q$, $q(0) = q_0$, and assuming that the initial level of control, X_0, is zero, the problem of maximizing expected net benefits of pollution control may be stated as

$$\max_u E_0 \int_0^\infty [v(t)\, Q(X(t)) - cu(t)] e^{-\delta t} dt \tag{4}$$

$$\begin{aligned} s.t.\ dq &= mdt + \sigma dz \\ \dot{X} &= u(t) \geq 0 \\ u(t) &\in [0,1] \\ X^0 &= 0 \ given \end{aligned}$$

where E_0 denotes expectation at $t = 0$ and δ is the positive discount rate. The fact that the rate at which pollution control is undertaken, $u(t)$, is greater than zero reflects the assumption that investment in pollution control is irreversible due to sunk capital costs. The value function for this problem may be written as

$$J(X, q) = \max E_t \int_t^{\infty} [v(\tau)Q(X(\tau)) - cu(\tau)]\, e^{-\delta\tau} d\tau. \tag{5}$$

The Hamilton-Jacobi-Bellman equation (H-J-B) is then

$$\delta J = \max_{u \geq 0} \left\{ e^{q}Q(X) - cu + \frac{1}{2}\sigma^2 J_{qq} + mJ_q + uJ_X \right\} \tag{6}$$

$$= e^{q}Q(X) + \frac{1}{2}\sigma^2 J_{qq} + mJ_q + \max_{u \geq 0} \left\{ u(J_X - c) \right\}$$

where the subscripts on J denote partial derivatives of the value function and $J_X > 0$ is the stochastic counterpart of the costate variable in deterministic dynamic programming. Note that the H-J-B equation is a second-order partial differential equation. Then, on the region where $J_X - c > 0$, it is optimal to control pollution at the highest possible level so that $u^* = 1$ (where * denotes optimal value). Thus we seek a solution to the differential equation

$$\frac{1}{2}\sigma^2 J_{qq} + mJ_q - \delta J = -e^{q}Q(X). \tag{7}$$

In general, partial differential equations are difficult to solve analytically. Fortunately, equation (7) is solvable. In fact, it can be treated as an ordinary differential equation because it does not contain a J_X term. The general solution to equation (7) is given by

$$J(X, q) = A(X)e^{\beta_1 q} + B(X)e^{\beta_2 q} + e^{q}Q(X)/\Delta \tag{8}$$

where $A(X)$ and $B(X)$ are arbitrary functions,

$$\Delta = \delta - m - \frac{1}{2}\sigma^2 = \delta - \alpha > 0, \tag{9}$$

and

$$\beta_{1,2} = [-m \pm \sqrt{m^2 + 2\delta\sigma^2}\ ] \ /\sigma^2 \tag{10}$$

are the positive and negative roots, respectively, of the characteristic equation $(\sigma^2/2)\beta^2 + m\beta - \delta = 0$.

As $q \to -\infty$, it must be optimal to stop accumulating control capital since the value of control goes to zero. This can only happen if $B(X) = 0$, i.e., the value function simplifies to

$$J\,(X, q) = A\,(X)e^{\beta_1 q} + e^{\,q}Q\,(Q(X)/\Delta. \tag{11}$$

On the boundary of the region of maximal control, necessary conditions for an optimum require $J_X - c = 0$ so that

$$A'\,(X)e^{\,\beta_1 q} + e^{\,q}Q'\,(X)/\Delta + c = 0. \tag{12}$$

It will also be the case that $J_{X_q} = 0$[1] implying

$$A'\,(X)e^{\,\beta_1 q}\beta_1 + e^{\,q}Q'\,(X)/\Delta = 0. \tag{13}$$

Solving for $B'(X)$, recalling that $v(t) = e^q$, substituting into equation (12), and solving for $Q'(X)$ will yield

$$vQ'\,(X) = \frac{\beta}{\beta - 1}\left(\delta - m - \frac{1}{2}\sigma^2\right)c. \tag{14}$$

Expression (14) says that the marginal instantaneous value of the service flow yielded by pollution control facilities must equal some multiple of marginal cost of pollution control. Notice that the sign of $\beta/(\beta - 1)$ is ambiguous. It is positive when $\beta \geq 0$ and negative otherwise.

Let now the region of maximal pollution control be denoted by ∂F which is a fixed curve in the control-valuation space (X,v). Using the implicit function theorem, it must hold on this boundary

$$\frac{dX}{dv}\,|_{\partial F} = \frac{-Q'\,(X)}{vQ''\,(X)} > 0. \tag{15}$$

1. This condition arises from the optimality of the maximal control boundary. The mathematical argument for this condition is quite complicated (for details, see WHITTLE 1983).

That is, the complete control boundary defines a monotonically increasing function, say $X^* = f(v)$. Then, if for the current pair (v,X), one has $X > X^*$, the stock of abatement capital (and thus pollution control) should not be increased at that instant. If, for current (v,X), one has $X^* \geq X$, then it is optimal to add the amount $X^* - X$ to the current stock of abatement capital. That is, we have a type of a *feedback* policy rule which specifies, for any pair (v,X), what level of control should be chosen. Thus at each decision point, both current and all future anticipated information are considered in choosing a particular level of control. To emphasize the point, the feedback policy rule is conditional on the current valuation of pollution control and reflects the fact that pollution control is not a now-or-never prescription, but intermediate levels of control may be optimal at any given point in time.

Thus the stock of abatement capital can only be increased but not decreased after the initial «jump». That is, if social valuations of the services yielded by the stock of abatement capital should increase in the future, it will be possible to increase pollution control. In the case, however, that social valuations would decline over time, the current stock of abatement capital would be higher than it would have been desired had the initial jump been lower.

The following Proposition summarizes our analysis so far and shows an important property of the maximal control boundary:

Proposition 1: Increases in the instantaneous variance of v (i.e., increasing uncertainty about the future valuation of pollution control) lead to a lower optimal stock of abatement capital (and thus pollution control) at any given v.

Proof: Suppose that $\beta \geq 0$. Now let $\Psi := \beta/(\beta - 1)(\delta - m - (\sigma^2/2))$. In the sequel, we first show that $\partial\Psi/\partial\sigma^2 < 0$. This follows from the fact that β satisfies the characteristic equation for the homogeneous form of (7), i.e., we have

$$\frac{1}{2}\sigma^2\beta^2 + (\alpha - \frac{1}{2}\sigma^2)\beta - \delta = 0.$$

Then, differentiating implicitely will yield

$$\frac{1}{2}\beta^2 + \sigma^2\beta\frac{\partial\beta}{\partial\sigma^2} + (\alpha - \frac{1}{2}\sigma^2)\frac{\partial\beta}{\partial\sigma^2} - \frac{1}{2}\beta = 0.$$

Solving for $\partial\beta/\partial\sigma^2$ and rearranging leads to $\partial\beta/\partial\sigma^2 \leq 0$. Now

$$\frac{\partial\Psi}{\partial\sigma^2} = (\delta - \alpha)\frac{\partial}{\partial\sigma^2}\left(\frac{\beta}{(\beta - 1)}\right) = (\delta - \alpha)\left(\frac{-\beta'}{(\beta - 1)^2}\right) \geq 0.$$

Then it follows from differentiating (14) with respect to σ^2

$$\frac{\partial X^*}{\partial \sigma^2} = \frac{\nu Q'(X)\dfrac{\partial \Psi}{\partial \sigma^2}}{\Psi \nu Q''(X)} \leq 0.$$

The kind of rule derived in Proposition 1 is familiar in other economic contexts, for example, in the wilderness conservation literature (see CLARKE and REED 1990). Although, in their paper the case of wilderness conservation is strengthened since the development of wilderness is irreversible, whereas in the framework considered here, investment in pollution control is supposed to be irreversible. Were environmental damages irreversible instead of investment in pollution control, the result of Proposition 1 would most likely be reversed.

3. THE CERTAINTY EQUIVALENCE APPROACH

What is the relationship between the type of optimal policy discussed so far and a «certainty equivalence» approach? Note first that a certainty equivalence approach requires to replace random variables (here: ν) by their expected values and then resolving the resulting deterministic problem. This is also called an *open-loop* approach where the environmental agency makes a pollution control decision at the outset and follows it regardless of any future information concerning the valuation of control benefits.[2] In contrast, the optimal closed-loop or feedback policy derived in section 2 was derived by assuming that additional information about the valuation of pollution control will be forthcoming. The difference between the optimal objective value using a stochastic dynamic programming approach and the objective value using certainty equivalence is usually called the «quasi-option value» (see, for example, ARROW and FISHER 1974, HENRY 1974, CONRAD 1980, MILLER and LAD 1984). Thus, if future information is forthcoming, there is a premium on those initial control decisions which preserve future flexibility, regardless of whether the decision maker is characterized by risk aversion or risk neutrality.

Let the expected value of $\nu(t)$ be denoted by $V(t)$. Then $V(t)$ can be calculated by noting that, from

$$dq = mdt + \sigma^2 dz,$$

2. Another possibility to model a policy in which the prospect of future information is disregarded would be an open-loop feedback approach where the decision maker updates his plans continuously as new information becomes available, but he also assumes that no *further* information will emerge as time goes by. The qualitative results, however, that we will derive below would still be true even under an open-loop feedback policy.

we have $q \sim N(q_0 + mt, \sigma^2 t)$. Hence, $v(t)$ has a log-normal distribution with expected value

$$V(t) = \exp\left\{q_0 + mt + \frac{\sigma^2}{2}t\right\} = v_0 e^{\alpha t}$$

where $\alpha = m + \sigma^2/2$. The deterministic problem is then

$$\max_{u \geq 0}\left\{\int_0^\infty [V(t)Q(X(t)) - cu(t)]e^{-\delta t}dt\right\} \tag{16}$$

$$\begin{aligned} s.t.\dot{X} &= u(t) \geq 0 \\ u(t) &\in [0,1] \\ X_0 &= 0 \textit{ given.} \end{aligned}$$

The Hamiltonian becomes

$$H = e^{-\delta t}[V(t)Q(X) - cu] + \lambda u$$

where λ is the costate variable. Since the Hamiltonian is linear in the control u, the solution involves «bang-bang control», with switching function

$$\sigma = -e^{-\delta t}c + \lambda = 0,$$

and costate dynamics

$$\dot{\lambda} = -e^{-\delta t}V(t)Q'(X).$$

If $\sigma(t) = 0$, then

$$\lambda = -e^{-\delta t}c$$

and

$$\dot{\lambda} = -e^{-\delta t}\delta c.$$

Simple calculations will yield

$$\delta c = V(t)Q'(X), \text{ or} \tag{17}$$

$$\delta c = v_0 e^{\alpha t} Q'(X). \tag{17'}$$

Let us compare equation (17) to the optimal policy rule derived in section 2 (see expression (14)). Note first that equation (17) results as the limiting form of the optimal policy when $\mu = 0$, $\sigma^2 \to 0$ and $\beta_1 \to -\infty$. It then follows from the fact that $\partial\Psi/\partial\sigma^2 \geq 0$ and $\partial X/\partial\sigma^2 \leq 0$ that using the certainty equivalence approach leads to a degree of control which is higher than it is optimal if investment in abatement capital is assumed to be irreversible. Furthermore, the higher the uncertainty, the bigger the difference between the optimal degree of pollution control and the degree of pollution control under the open-loop policy.

How can the concept of quasi-option value be defined in the infinite horizon framework considered here? As it has already been mentioned, it is simply the difference between the values of the objective function (4), using the optimal policy, and the open-loop certainty equivalence approach. Using this definition, it then follows from the sub-optimality of the certainty equivalence approach that the quasi-option value must be non-negative. Thus the expected gains from learning about future valuations of pollution control benefits are not taken into account by using the certainty equivalence approach. If investments in pollution control are irreversible, it is optimal to wait for forthcoming information. Proposition 2 summarizes these results:

Proposition 2: (*i*) The stock of abatement capital (i.e., the degree of pollution control) using the open-loop certainty equivalence approach is higher than under the closed-loop approach, and, furthermore, (ii) the higher the future uncertainty, the bigger the difference between the optimal closed-loop policy and the open-loop policy.

The lower level of pollution control under a closed-loop policy has at least two advantages: (1) If future valuation of control benefits are higher than expected, one can invest more in pollution control later in the future (provided that environmental damages are not irreversible by that point). (2) By controlling less today, the level of regret will be lower if future net benefits turn out to be lower than expected since investment in pollution control cannot be retrieved for other purposes.

3. THE CASE OF A CRITICAL PERIOD

So far, we have assumed that the evolution of the social valuation of pollution control benefits can be described as a continuous stochastic process. Suppose now that, at some point in the evolution of the ecosystem, if the policy maker does not intervene and control pollution at that instant, it could never be optimal for him to control pollution subsequently. This might happen if continued stress on an ecosystem triggers irreversible changes after a certain threshold level is hit. Or, to put it differently, the damage function of a particular pollutant may become discontinuous at some point in time and take a «jump». This may capture a situation where a major environmental catastrophe will take place at an unknown point in the future. In general, such discontinuities may occur if the probability of an environmental hazard is independent, or depends discontinuously, on the current pollution stock. Following FISHER and HANEMANN (1989), we call a point in time, abusing terminology a little bit, with this property a «*critical period*»[3]. Of course, whether such a phenomenon exists and what conditions bring it about depends on the specifics of the ecosystem structure. In our framework, we shall model possible discontinuities by assuming that, if this event happens, control benefits drop to zero and remain there forever.

As an illustration, consider the case of stock pollutants such as greenhouse gases. Increasing atmosphere CO_2 concentrations may cause drastic climatic changes once they reach sufficiently high unknown levels, inducing environmental effects such as extinction of certain species, inundations of coastal regions, and so forth. If this is the case, it will never pay to cut back on CO_2 emissions afterwards[4]. Since concentration of greenhouse gases have a residence time in the atmosphere that is measured in decades, even centuries, a failure to reduce emissions before a certain point in time can safely to be assumed to be irreversible. Even under the extreme and unrealistic assumption that climate change is reversible in the short-term, important impacts of the elevated concentrations such as the above mentioned loss of species, inundations of coastal regions, etc. would be irreversible (for further discussion, see FISHER and HANEMANN 1993).

We model the possibility that pollution control might become useless after a certain point in time by introducing a *Poisson* process[5]. A Poisson process is a stochastic process subject to jumps of fixed or random size, for which the arrival times follow a Poisson distribution. The jumps are often called «events» in the literature. We now allow for the possibility that, at some random point in time, the valuation of control benefits, v, will take a jump downwards. More specifically, to modify our basic model, we assume that $v(t)$ follows the mixed Brownian motion/jump process

$$\frac{dv}{v} = \alpha dt + \sigma dz - dw, \quad \text{where } dw = \begin{cases} 0, \text{ with } prob \; 1-\lambda dt \\ 1, \text{ with } prob \quad \lambda dt. \end{cases} \tag{2'}$$

dw is the increment of a Poisson process with mean arrival rate λ, and dw and dz are independent so that $E(dzdw) = 0$. Suppose that, if an event occurs, w falls by 1 with probability 1 (i.e., the jump size is 1 for sure). Equation (2′) says that v will fluctuate as a geometric Brownian motion process but for each time interval dt there is a small

3. In contrast to FISHER and HANEMANN (1989), in the model considered here, the occurence of a critical period is subject to a stochastic process. In their paper, it is known with certainty which period is critical.
4. Note that, strictly speaking, it is technically not possible to abate CO_2 emissions. The only way to reduce CO_2 emissions is to burn less fossil fuels.
5. For a quite comprehensive discussion of Poisson events in economic contexts, see MERTON (1971).

probability λdt that it will suddenly drop to zero, and remain there forever.[6] Thus the occurrence of a Poisson event induces the process to stop, since zero is a natural absorbing barrier for a geometric Brownian motion process.

The partial differential equation (7) will now be modified to

$$\frac{1}{2}\sigma^2 J_{qq} + mJ_q - J(\delta + \lambda) = -e^{q}Q(X). \tag{7'}$$

The solution of (7′) is again of the form (8), but now β_2 is the negative root to the slightly different characteristic equation $(\sigma^2/2)\beta^2 + m\beta - (\delta + \lambda) = 0$. The negative solution is then

$$\beta_2 = (-m - \sqrt{m^2 + 2(\delta + \lambda)\sigma^2})\ /\sigma^2. \tag{10'}$$

From equation (10′), we may conclude that the Poisson parameter λ gets added to the discount rate δ. Since $\partial\Psi/\partial\delta = \beta_2/(\beta_2 - 1) > 0$, the effect on the control level will be negative and, thus, the occurrence of a Poisson event *lowers* the optimal degree of pollution control:

Proposition 3: The occurrence of a «critical period», modelled as a (time-independent) Poisson event, leads to a lower stock of abatement capital (i.e., lower degree of pollution control) than otherwise would have been optimal.

Proposition 3 is in contrast to FISHER and HANEMANN (1989) who claim that the existence of a critical period would always strengthen the case for controlling pollution. It might also seem that this result is somewhat counterintuitive. One would expect that the considerable risk of a period after which it never pays to control pollution, would lead to a higher degree of pollution control now. That this conjecture does not need to be true in general has been shown, however, in related economic contexts (see, for example, CLARKE and REED 1994).

The reason for this result, in the model considered here, might be that the occurrence of the Poisson event is largely «unavoidable» or «policy-independent», so there is little incentive to increase the degree of pollution control now. This is to say, the presence of a Poisson event makes the future even more uncertain, so to maximize net benefits it makes sense to discount future benefits heavily.

On the other hand, if the occurrence of a critical period could be influenced, or, even postponed, by taking control measures now, then it might be the case that the degree of

6. We could relax this assumption and allow for the possibility that v drops less than 100% of its original value. This would, however, complicate the analysis without providing further economic insight.

pollution control is higher than in the corresponding world without such a risk.[7] Indeed, CLARKE and REED (1994) prove such a result in a different framework where uncertainty about the future is captured by a «hazard function» reflecting the probability of a major catastrophe.[8] In any case, the possibility of a critical period leads to ambiguous results as to whether control measures should be increased or decreased in the presence of irreversible ecosystem damages.

4. CONCLUSIONS

This paper has applied stochastic dynamic programming involving geometric Brownian and Poisson processes to environmental decision making. The main concern was the optimal timing of pollution control decisions in a fairly general economic setting. It has been shown that future uncertainty, if it is combined with irreversibility due to sunk capital costs, tends to lower the optimal degree of pollution control, provided the policy maker takes into account the value of forthcoming information. Thus it is optimal to use a feedback or closed-loop policy and not an open-loop approach which disregards the prospect of future information. This result is due to the assumption that investment in control facilities is irreversible (i.e., they cannot be used for another purpose). In such a situation, waiting to invest is optimal. Of course, if environmental damage itself is irreversible, our result would most likely change in the sense that controlling more today would be the optimal solution.

It has also been shown, however, that the assumption of a «critical period» after which it never pays to control, modelled as the possible occurrence of a time-independent Poisson event, may reduce the degree of pollution control because the decision maker wants to discount future benefits of pollution control more heavily. This result might be changed if the decision maker could somehow influence the probability of such a discontinuity to occur. This assertion, however, remains to be proven in a formal model, for example, by using a time-dependent Poisson process.

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7. This would imply that the probability of a Poisson event depends on the control decision in the sense that a higher degree of control would lower the probability of a sudden jump of v.
8. CROPPER (1976) also analyzes the possibility of environmental catastrophes that reduce social utility to zero. He assumes that the probability of a catastrophe is dependent on the size of a given pollution stock. CROPPER, however, provides no qualitative insight between catastrophic risk and actual pollution management.

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SUMMARY

This paper sets up a model using stochastic dynamic programming to analyze pollution control decisions. Environmental decision making is characterized by both uncertainty and irreversibility: Uncertainty might be related to future costs and benefits of adopting a particular control policy. Irreversibility may arise when investments in abatement capital are irreversible, or when environmental damage becomes irreversible at a certain point in time. Under such circumstances, the question of the optimal timing of pollution control arises. This paper shows that: (1) Increases in the uncertainty of future control benefits lead to a lower current degree of pollution control if investment in control facilities is irreversible, (2) ignoring the possibility of new information in the future leads to a higher level of control than it is optimal, and (3) the possibility of a major catastrophe may result in a lower optimal degree of pollution control.

ZUSAMMENFASSUNG

Diese Arbeit beschreibt ein Modell zur Analyse von Umweltschutzmassnahmen unter Verwendung von stochastischer dynamischer Programmierung. Umweltschutzentscheidungen sind sowohl durch Unsicherheit als auch durch Irreversibilität gekennzeichnet. Unter diesen Umständen stellt sich die Frage nach dem optimalen Zeitpunkt von Umweltschutzmassnahmen. Die Ergebnisse der vorliegenden Arbeit lassen sich wie folgt zusammenfassen: (1) Zunehmende Unsicherheit über den künftigen Nutzen von Umweltschutzmassnahmen reduziert diese, falls Investitionen in Umweltschutzanlagen irreversibel sind. (2) Falls die Möglichkeit neuer, künftiger Informationen ignoriert wird, resultieren Umweltschutzmassnahmen, die im Vergleich zum Optimum zu hoch sind. (3) Die Möglichkeit einer Katastrophe kann dagegen zu geringeren Umweltschutzmassnahmen führen als im Vergleich zum Optimum nötig wären.

RESUME

Cet article propose un modèle utilisant la programmation dynamique stochastique pour analyser les décisions de protection de l'environnement. Ces décisions sont charactérisées par l'incertitude et l'irreversibilité. Dans ces circonstances la question du moment optimal de la protection de l'invironnement se pose. Cet article démontre que: (1) Une augmentation de l'incertitude des bénéfices futurs de la protection de l'invironnement conduit à un niveau plus bas de pollution courante si les investissements dans la protection de l'invironnement sont irréversible, (2) la non prise en compte de nouvelles informations dans le futur conduit à un niveau plus élevé de protection que dans la situation optimale, (3) la possibilité d'une catastrophe majeure peut résulter dans un niveau plus bas de protection.