The Geographical Composition of Swiss Foreign Trade: A Semiflexible Approach

ULRICH KOHLI*

1. INTRODUCTION

The first objective of this paper is to model the Swiss demand for imports and supply of exports by regions of origin and destination. We concentrate our attention on trade with industrialized nations. We consider nine foreign regions which together cover the entire OECD area[1].

Table 1 reports the 1958 and 1987 import and export shares (in percentage points) of the nine regions under consideration[2]. It is obvious that Germany is, and has long been, Switzerland's main trading partner. In recent years, German goods have made up nearly forty percent (in value) of Swiss imports from the OECD area. On the export side, Germany purchases just over one quarter of Switzerland's exports to the industrialized world. Other European countries, particularly France and Italy, make up much of the balance. Nevertheless, the share of non-European industrialized countries is significant. This is particularly true on the export side, where sales to the United States, Canada, Japan, Australia, and New Zealand represent about one sixth of all Swiss exports to the OECD area. It is also apparent from Table 1 that the geographical distribution of Swiss external trade is not constant through time. Comparing 1958 with 1987 figures, one notices the large increase over the last three decades in the German shares of Swiss imports and exports. One also notices the five-fold increase in Japan's share on the import side, and the almost three-fold increase on the export side. North America's share of Swiss imports has fallen dramatically, although Switzerland's exports to that region have been less affected. Also noteworthy are the relatively large declines in the Benelux and other-OECD shares of Swiss exports.

* Department of Economics, University of Geneva, 102, boulevard Carl-Vogt
CH-1211 Geneva 4, Switzerland.
phone: (41-22) 705-8285, fax: (41-22) 705-8293, e-mail: «Ulrich.Kohli@ecopo.unige.ch»
Financial support from the Swiss National Science Foundation is gratefully acknowledged.

1. In 1987, OECD trade accounted for 90.2% and 79.3% of Swiss merchandise imports and exports, respectively.
2. See Section 6 for a description of the data. This study being part of a larger project, the sample period was taken to be the same as in KOHLI (1994).

Following ARMINGTON (1969), the way the geographical distribution of a country's foreign trade is usually modeled is by estimating a CES (CET) import (export) allocation model[3]. This approach, which assumes that imports and exports are separable from domestic inputs and outputs, yields import demand (export supply) functions which are conditional on the quantity of aggregate imports (exports). Such models have become extremely popular in recent years with the construction of world trade models and computable general equilibrium models. However, these models suffer from the drawback that – by definition – the CES (CET) functional form constrains the elasticities of substitution (transformation) between any pair of goods to be identical and constant through time. Not only is this unrealistic, particularly if the number of goods is large, but it also rules out complementarity relationships. It therefore seems much preferable to use a flexible functional form in order to avoid these shortcomings.

Unfortunately, the use of flexible functional forms is not necessarily foolproof either. Indeed, the number of parameters increases rapidly with the number of goods under consideration. This may make the estimation of large models unpractical, and it obliges the researcher to make some painful choices. Thus, one can, for instance, use the separability hypothesis repeatedly in order to contain the number of parameters to be estimated at each stage of the disaggregation. This approach, used by KOHLI (1985) in conjunction with the Translog functional form, is unavoidably somewhat arbitrary. An alternative approach destined to limit the number of parameters is to use the characteristics approach of KOHLI and MOREY (1988). While the number of parameters is then independent of the number of goods in the sample, this model too rests on the CES functional form.

Yet another possibility is to use a semiflexible functional form as proposed by DIEWERT and WALES (1988b) in the context of consumer theory. This is the approach which we will follow here, and the second objective of this paper is to illustrate how semiflexible functional forms can be used when modeling rather large demand or supply systems. The use of semiflexible functional forms allows for large savings in terms of the number of parameters to estimate. This may reduce problems of multicollinearity, and it may translate into substantial savings in terms of computing costs. Moreover, it allows one to envisage the estimation of still larger models.

The remainder of this paper is organized as follows. We briefly review the two-stage budgeting procedure in the next section. Section 3 presents the Symmetric Normalized Quadratic functional form which we will use in our empirical work. Sections 4 and 5 investigate curvature conditions and the concept of semiflexibility. Section 6 discusses the empirical implementation of the model, Section 7 presents the empirical results, and Section 8 concludes.

3. The CES (constant elasticity of substitution) functional form is due to ARROW et al. (1961), while the CET (constant elasticity of transformation) has been proposed by POWELL and GRUEN (1968).

2. TWO-STAGE BUDGETING

For expository purposes, we find it convenient to view imports as inputs to the technology, and exports as outputs[4]. We assume a large number of profit-maximizing firms which choose their input and output mix for given input and output prices, subject to their technology. It is well known that under constant returns to scale, the competitive equilibrium can be represented as the solution to the problem of maximizing GNP subject to domestic factor endowments, and for given output and variable input prices[5]. This problem yields a system of disaggregate import demand and export supply functions, together with the (direct or inverse) demand and supply functions for the domestic factors and goods. If the number of imports, exports, domestic factors and domestic outputs is large, however, this problem rapidly becomes unmanageable from an empirical perspective. To simplify matters, assume that imports as well as exports are weakly separable from the remaining inputs and outputs. The profit-maximizing problem can then be decomposed into two parts. In the first stage, importers determine their optimum import mix by minimizing import costs, subject to the import aggregator function, and for given import prices. At the same time, exporters select their export mix by maximizing export revenues, subject to the export aggregator function, and for given export prices. In the second stage, firms establish their optimum level of aggregate imports and exports together with their demand and supply of domestic inputs and outputs by maximizing their profits for given input (including aggregate import) and output (including aggregate export) prices, subject to their technology. The second stage of this optimization process is well known and has been studied extensively in the literature with the estimation of aggregate import and export functions[6]. The focus of this paper is on the first stage of this twin-optimization problem.

Assume that there are J types of imports (e.g. imports from J different regions). Let $x_M \equiv [x_{M_j}]$ and $w_M \equiv [w_{M_j}]$ be the vectors of import quantities and prices, respectively; y_M and p_M denote the quantity and the price of aggregate imports, respectively. Let the import aggregator function be given by the following:

$$y_M = f(x_M). \tag{1}$$

4. Note, however, that the two-stage budgeting approach is also compatible with consumer theory; see DEATON and MUELLBAUER (1980), for instance.
5. This describes the GNP function approach to modeling imports and exports; see KOHLI (1978, 1991) and WOODLAND (1982).
6. For Swiss estimates, see KOHLI (1992).

We assume that $f(\cdot)$ is increasing, linearly homogeneous, and concave. Assuming cost minimization, the import aggregation technology can also be described by the following import cost function:

$$C(y_M, w_M) \equiv \min_{x_M} \left\{ \Sigma_j w_{M_j} x_{M_j} : f(x_M) \geq y_M \right\}. \tag{2}$$

This cost function is linearly homogeneous in import prices, and in the quantity of aggregate imports. This last property enables us to write:

$$C(y_M, w_M) = c(w_M) y_M, \tag{3}$$

where $c(\cdot)$ is the unit cost function and, under competitive conditions, it is equal to p_M.

The demand for import component j can then be derived by differentiation, a result implied by SHEPHARD's (1953) Lemma[7]:

$$x_{M_j} = \frac{\partial C(y_M, w_M)}{\partial w_{M_j}} = \frac{y_M \partial c(w_M)}{\partial w_{M_j}}. \tag{4}$$

It is apparent from (4) that the demand for import component j is conditional on the level of aggregate imports, y_M.

The unit cost function can be used to define the matrix of ALLEN-UZAWA elasticities of substitution between import components, $\Sigma \equiv [\sigma_{jk}]$. It is well known that Σ can be obtained directly from the unit cost function (UZAWA, 1962):

$$\sigma_{jk} = \frac{c(w_M) c_{jk}(w_M)}{[c_j(w_M) c_k(w_M)]}, \tag{5}$$

where $c_j(w_M) \equiv \partial c(w_M)/\partial w_{M_j}$ and $c_{jk}(w_M) \equiv \partial^2 c(w_M)/(\partial w_{M_j} \partial w_{M_k})$. Furthermore, we may define the partial price elasticities of demand:

$$\xi_{jk} \equiv \frac{\partial \ln[x_{M_j}(y_M, w_M)]}{\partial \ln(w_{M_k})}.$$

7. See DIEWERT (1974), for instance.

Making use of (4) and (5), it is apparent that these can be calculated from the estimate of Σ directly:

$$\xi_{jk} = \sigma_{jk} s_k , \tag{6}$$

where s_k is the value share of import component k in total imports. Note that these elasticities are only partial elasticities. That is, they are conditional on the level of total imports. A change in the price of import component k will, however, lead to a variation in the demand for aggregate imports since, as shown by (3), it will impact on p_M. Total price effects can be calculated as follows:

$$\varepsilon_{jk} = \xi_{jk} + s_k \varepsilon_{MM} ,$$

where ε_{MM} is the own price elasticity of the demand for imports suitably defined.

On the export side, the revenue-maximization problem that makes up the first step of the two-stage budgeting problem is the mirror image of the cost-minimization problem which we just discussed. Assume that there are I types of exports (exports to I different regions). We denote the vector of their quantities by $y_X \equiv [y_{X_i}]$, and the corresponding price vector by $p_X \equiv [p_{X_i}]$; x_X and w_X represent the quantity and the price of aggregate exports, respectively. Let the export aggregator function (or factor requirements function) be given by $h(\cdot)$. We assume that $h(\cdot)$ is increasing, linearly homogeneous, and convex:

$$x_X = h(y_X). \tag{7}$$

Assuming revenue maximization, the export aggregation technology can also be described by the following revenue function:

$$R(p_X, x_X) \equiv \max_{y_X} \left\{ \Sigma_i p_{X_i} y_{X_i} : \mathrm{h}(\mathrm{y}_X) \leq x_X \right\}. \tag{8}$$

This revenue function is linearly homogeneous and convex in export prices, and linearly homogeneous in the quantity of aggregate exports. Thus:

$$R(p_X, x_X) = r(p_X) x_X , \tag{9}$$

where $r(\cdot)$ is the unit revenue function and it is equal to w_X.

The supply of export component i can be obtained by differentiation of (9), a consequence of HOTELLING's (1932) Lemma:

$$y_{X_i} = \frac{\partial R(p_X, x_X)}{\partial p_{X_i}} = \frac{x_X \partial r(p_X)}{\partial p_{X_i}}. \tag{10}$$

As indicated by (10), the supply of export component i is conditional on the level of aggregate exports, x_X.

The unit revenue function can be used to define a set of elasticities of transformation between export components, Θ_{ih}. These can be obtained from the unit revenue function directly:

$$\Theta_{ih} = \frac{r(p_X)r_{ih}(p_X)}{[r_i(p_X)r_h(p_X)]} \tag{11}$$

where $r_i(p_X) \equiv \partial r(p_X)/\partial p_{X_i}$ and $r_{ih}(p_X) \equiv \partial^2 r(p_X)/(\partial p_{X_i} \partial p_{X_h})$. Next, we can define a set of price elasticities of supply:

$$\psi_{ih} \equiv \frac{\partial \, ln[y_{X_i}(p_X, x_X)]}{\partial \, ln(p_{X_h})}.$$

Moreover, it follows from (10) and (11) that:

$$\psi_{ih} = \Theta_{ih} v_h, \tag{12}$$

where v_h is the value share of export h in total exports. These elasticities, like their import counterparts, are partial elasticities. That is, they are conditional on the level of total exports. Total price elasticities can be calculated in the following way:

$$\eta_{ih} = \psi_{ih} + v_h \eta_{XX},$$

where η_{XX} is the own price elasticity of the supply of exports appropriately defined.

Let us finally point out that the approach described in this section allows for the estimation of true ALLEN-UZAWA partial elasticities of substitution or transformation (i.e. along an isoquant or a production possibilities frontier) between imports or exports of different nature without the difficulties that have plagued much of the empirical work in this area; see LEAMER and STERN (1970), for instance. This is true independently of the number of components under consideration. Moreover, as long as one uses a flexible or semiflexible functional form, this procedure is considerably more general than the one usually followed in the literature, where most researchers have opted for the CES or the CET model.

3. THE SYMMETRIC NORMALIZED QUADRATIC AGGREGATOR FUNCTION

We use the Symmetric Normalized Quadratic functional form introduced by DIEWERT and WALES (1987, 1988a) to approximate the import and export price aggregator functions. Allowing the technology to shift over time, the unit import price aggregator function can be written as follows (t is a time index):

$$p_M = \frac{\tfrac{1}{2}\Sigma\Sigma a_{jk} w_{M_j} w_{M_k}}{\left(\Sigma\alpha_j w_{M_j}\right)} + \Sigma c_j w_{M_j} + \Sigma\gamma_{jt} w_{M_j} t + \tfrac{1}{2}\gamma_{tt}\left(\Sigma\alpha_j w_{M_j}\right)t^2 \quad (13)$$

$$j, k = 1, ..., J,$$

where $\Sigma_k a_{jk} = 0$ and $a_{jk} = a_{kj}$. The α_j's are nonnegative and predetermined subject to the condition $\Sigma\alpha_j = 1$. The denominator of the first term on the right-hand side of (13), $\Sigma\alpha_j w_{Mj}$,can thus be viewed as a fixed-weighted input price index. Unlike the better known Normalized Quadratic also proposed by DIEWERT and WALES (1987), the Symmetric Normalized Quadratic treats all inputs in the same way. This functional form is flexible at the point of normalization of the data; furthermore, using the terminology of DIEWERT and WALES (1992), we can add that it is also flexible with respect to time, not merely TP flexible. The Symmetric Normalized Quadratic cost function is necessarily homogeneous of degree one in prices, and it is globally concave if and only if $A \equiv \left[a_{jk}\right]$ is negative semidefinite. Note that, given that $\Sigma_k a_{jk} = 0$, A is at most of rank $J-1$. In what follows, we will find it convenient to define $\tilde{A}$ as the matrix that we obtain by deleting the last row and the last column of A.

As indicated by (4), we can derive the disaggregate import demand functions by differentiation. It is convenient to express them relative to aggregate imports:

$$\frac{x_{M_j}}{y_M} = \frac{\Sigma a_{jk} w_{M_k}}{\left(\Sigma\alpha_k w_{M_k}\right)} - \frac{\tfrac{1}{2}\alpha_j \Sigma\Sigma a_{kn} w_{M_k} w_{M_n}}{\left(\Sigma\alpha_k w_{M_k}\right)^2} + c_j + \gamma_{jt} t + \tfrac{1}{2}\gamma_{tt}\alpha_j t^2 \quad (14)$$

$$j, k, n = 1, ..., J.$$

The revenue (or export price aggregator) function is similar to (13), except that the notation is modified to reflect the fact that we are dealing with a unit revenue function:

$$w_X = \frac{\frac{1}{2}\Sigma\Sigma b_{ih} p_{X_i} p_{X_h}}{\left(\Sigma\beta_i p_{X_i}\right)} + \Sigma d_i p_{X_i} + \Sigma\delta_{it} p_{X_i} t + \tfrac{1}{2}\delta_{tt}\left(\Sigma\beta_i p_{X_i}\right) t^2 \qquad (15)$$

$$i, h = 1, ..., I,$$

where $\Sigma_h b_{ih} = 0$ and $b_{ih} = b_{hi}$. The β_i's are nonnegative and predetermined subject to the condition $\Sigma\beta_i = 1$. This function is flexible at the point of normalization of the data, it is necessarily linearly homogeneous, and it is convex if and only if B is positive semidefinite. B is at most of rank $I - 1$, and we define $\tilde{B}$ as the matrix obtained by deleting the I^{th} row and column of B. The disaggregate export supply functions are derived relative to aggregate exports:

$$\frac{y_{X_i}}{x_X} = \frac{\Sigma b_{ih} p_{X_h}}{\left(\Sigma\beta_h p_{X_h}\right)} - \frac{\frac{1}{2}\beta_i \Sigma\Sigma b_{hm} p_{X_h} p_{Xm}}{\left(\Sigma\beta_h p_{X_h}\right)^2} + d_i + \delta_{it} t + \tfrac{1}{2}\delta_{tt}\beta_i t^2 \qquad (16)$$

$$i, h, m = 1, ..., I.$$

4. CURVATURE CONDITIONS

As noted by DIEWERT and WALES (1987), one of the most frustrating problems confronting applied economists estimating flexible functional forms is that the estimates very often fail to satisfy required curvature conditions. Yet, these curvature conditions are implied by economic theory and they must be satisfied for the estimates to be meaningful. Curvature conditions can sometimes be imposed in empirical work. In some cases, e.g. when one uses Cobb-Douglas or Leontief functional forms, the curvature conditions are imposed automatically and the question does not even arise. In the Translog case, convexity or concavity can be imposed locally if needed. However, there is no guarantee then that the estimated function will satisfy these conditions globally, and the analyst must check for each observation whether or not the curvature conditions are met. Curvature conditions can sometimes be imposed globally, but the flexibility of the functional form may be destroyed in the process.

Fortunately, convexity or concavity can be imposed globally in the case of the family of Normalized Quadratic functional forms without endangering their flexibility. As noted above, the Symmetric Normalized Quadratic function form is globally concave (convex) if and only if A is negative (positive) semidefinite. This restriction can be imposed in empirical work if needed through a reparameterization of the estimating functions. For this purpose, we use the technique originally proposed by WILEY, SCHMIDT, and BRAMBLE (1973). As shown by these authors, a sufficient condition for a NxN matrix Z to be positive semidefinite is that it can be written as:

$$Z = TT' \tag{17}$$

where $T \equiv [\tau_{mn}]$ is a lower triangular matrix. DIEWERT and WALES (1987), exploiting a result by LAU (1978), have shown that the above condition is also necessary for Z to be positive semidefinite. Thus, we define:

$$T \equiv \begin{bmatrix} \tau_{11} & 0 & \cdots & 0 \\ \tau_{21} & \tau_{22} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \tau_{N1} & \tau_{N2} & \cdots & \tau_{NN} \end{bmatrix}.$$

We then get:

$$Z = \begin{bmatrix} \tau_{11}^{2} & \tau_{11}\tau_{21} & \cdots & \tau_{11}\tau_{N1} \\ \tau_{11}\tau_{21} & \tau_{21}^{2} + \tau_{22}^{2} & \cdots & \tau_{21}\tau_{N1} + \tau_{22}\tau_{N2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \tau_{11}\tau_{N1} & \tau_{21}\tau_{N1} + \tau_{22}\tau_{N2} & \cdots & \tau_{N1}^{2}\tau_{N2}^{2} + \cdots\tau_{NN}^{2} \end{bmatrix}.$$

This reparameterization ensures that Z is convex. Similarly, a matrix Z is negative semidefinite if it can be written $Z = -TT'$. Applying this result to $\tilde{A}$ and $\tilde{B}$ in (13) and (15) guarantees that the Symmetric Normalized Quadratic cost and revenue functions are, respectively, concave and convex in prices.

5. SEMIFLEXIBILITY

One difficulty with flexible functional forms is that the number of parameters increases very rapidly with the number of goods. Even if one assumes linear homogeneity, the number of parameters in a N-good model with technological change is equal to $2N + 1 + (N - 1)\ N/2$. Thus, if $N = 2$, the number of parameters is 6. If $N = 4$, there are 15 parameters, and if $N = 8$, the number of parameters rises to 45. If $N = 16$, the number of parameters jumps to 153! This large number of parameters may lead to serious problems of multicollinearity, and it might even make it impossible to estimate the model unless the number of observations is unusually large. Moreover, it makes the estimation very awkward and time consuming. Does this mean that flexible functional forms have to be abandoned, and that we are back with very restrictive functions such as the CES/CET? Not necessarily. One way to get around the problem has been suggested by DIEWERT and WALES (1988b) who introduce the concept of K-flexibility, or semiflexibility.

DIEWERT and WALES define a functional form $g(z)$ as semiflexible at some point z^* if it has enough free parameters for $g(z^*)$, its gradient $\nabla g(z^*)$, and its Hessian $\nabla^2 g(z^*)$ to attain arbitrary values, provided that $\nabla 2g(z^*)$ is restricted to have rank K < N. A semiflexible functional form therefore has less parameters than a flexible one, and, although it is not flexible, it does not impose any obvious *a priori* restrictions on the size or the signs of the elasticities of substitution or transformation.

The concept of semiflexibility rests on some new results obtained by DIEWERT and WALES and which apply to semidefinite matrices. Central to their demonstration is the result that any positive semidefinite NxN matrix Z of rank K, where K is less than the maximal possible rank N, has a K-column triangular decomposition. This means that (17) is still valid, except that now $\tau_{mn} = 0$ for $1 \leq m < n \leq N$ and for $n = K + 1, ..., N$. Thus, T is a lower triangular N by N matrix which has zeros in its last $N - K$ columns. Note that T (and hence Z) has only $N(N + 1)/2 - (N - K)(N - K + 1)/2$ free parameters. This may imply a substantial reduction in the number of parameters to be estimated. For instance, if $N = 8$ and $K = 3$, the number of free parameters is 21, as opposed to 36 in the unrestricted case. The reparameterization (17) applied to $-\tilde{A}$ and $\tilde{B}$ in (13) and (15) yields the K-flexible, or semiflexible, version of the Symmetric Normalized Quadratic cost and revenue functions, thereby representing a potentially large saving in terms of free parameters. Moreover, this reparameterization guarantees that the correct curvature conditions will be met globally. The idea behind the DIEWERT and WALES procedure is that the researcher will chose K between 1 and N, depending on computational feasibility and degrees of freedom in the model. Needless to say, if $K = 0$ all flexibility is lost: the cost and revenue functions become linear, and the underlying technologies reduce to the Leontief form.

The assumption that $K < N$ may sound very restrictive. However, it need not be in practice. Indeed, it very often turns out in empirical work that many of the τ_{mn}'s are very close to zero anyway. Recent empirical results by KOHLI (1994) suggest that some reduction in flexibility does not do any injustice to the data. Our empirical work is therefore conducted on the basis of a value of K equal to 4, a fairly conservative level which still allows for considerable amounts of flexibility, but which nevertheless greatly facilitates the estimation by reducing the number of parameters by ten compared to the fully flexible version of the model ($K = 8$).

6. DATA AND ESTIMATION TECHNIQUE

We consider Swiss imports from, and exports to, twenty-four OECD-member nations. Special attention is devoted to the European trading partners of Switzerland. The disaggregation which we adopt for both imports and exports is the following one:

1. United States and Canada
2. Japan
3. EFTA (Austria, Finland, Iceland, Norway, Sweden)
4. United Kingdom
5. Germany
6. France
7. Italy
8. Benelux (Belgium, Netherlands, Luxembourg)
9. Other OECD (Australia, New Zealand, Denmark, Greece, Ireland, Portugal, Spain, Turkey, Yugoslavia).

All data are annual for the period 1958 to 1987. Import and export data, in current U.S. dollar terms, are drawn from *Direction of Trade Statistics.* Import prices are proxied by foreign export unit values, and they are taken from *International Financial Statistics.* Export prices, proxied by foreign import unit values, are drawn from the same source. All prices are expressed in U.S. dollars, and they are normalized to one for 1980[8]. In the case of regions 1, 3, 8 and 9, aggregation is carried out by computing Tornqvist price indexes of the disaggregate country series. However, no price series are available for Portugal, Iceland, Turkey and Yugoslavia so that these countries are left out in the calculation of the corresponding regional price indexes. The time index (t) is defined as a trend with unit annual increments, and normalized to zero for 1980.

We assume that the demand and supply functions (14) and (16) are random due to errors in optimization. The model is estimated by using an iterative version of Zellner's method for seemingly unrelated regressor equations as implemented in SHAZAM (WHITE, 1978), version 7.0. This method is numerically equivalent to maximum likelihood. Initial results revealed a severe problem of first-order autocorrelation. Estimation of the system of import and export equations, allowing for first-order autocorrelation and constraining the autocorrelation coefficients to be the same in all equations[9], produced estimates of the autocorrelation coefficients very close to unity. The model was therefore reestimated in first-difference form. That is, if we rewrite equations (14) more compactly as $x_M/y_M = \emptyset\ (w_M, t, A, c, \gamma)$, the model was estimated as $\Delta\ (x_M/y_M) = \Delta\emptyset\ (w_M, t, A, c, \gamma) + u$, where u is a vector of additive disturbances which we assume to be identically distributed,

8. Since both the cost function and the revenue function are linearly homogeneous in prices, the derived demand and supply functions are homogeneous of degree zero in those variables, i.e. only relative prices matter. There would therefore be nothing gained in converting the data into Swiss francs.

9. This procedure ensures that adding-up is still verified; see BERNDT and SAVIN (1975).

serially independent, normal random vectors with mean vector zero. The same treatment applies to the export model. The differencing procedure makes it impossible to identify the first-order terms (c_j's and d_i's). This is not a serious concern, however, since we are primarily interested in obtaining estimates of the Hessian matrices of the cost and revenue functions, and these do not contain those constant terms. In each model, we have 261 observations (nine equations times 29 annual observations) to estimate 36 unknown parameters[10]. All estimations were carried out on a VAX 8700 computer.

7. ESTIMATION RESULTS

Parameter values for the import model are reported in Table 2 after the α_j's were set to 1980 shares for estimation purposes[11]. Asymptotic-t values are shown in parentheses[12]. Estimates of the partial price elasticities of import demand (1987 values) are reported in Table 3[13]. Asymptotic-t values are again shown in parentheses[14]. It appears that many of these elasticities are determined with quite a high degree of precision, with about one quarter of the t values in excess of two in absolute value, and over half in excess of unity.

Looking first at the own price elasticities (ξ_{jj}), it appears that imports from the United Kingdom, EFTA, Italy, and France are the most price responsive. Imports from the Benelux, the United States and Canada, Japan, and the «other-OECD» category, on the other hand, are rather price inelastic.

Regarding the cross price elasticities (ξ_{jk}), one notes the strong – and statistically significant – complementarity between imports from North America and imports from the United Kingdom, as well as between imports from EFTA and those from the other-OECD category. A number of additional complementarity relationships implicate North America, Japan, the United Kingdom, and the smaller OECD nations, although they do not appear to be statistically significant. Fairly strong substitution effects can be detected between imports from Japan and imports from EFTA and small OECD countries; between imports from the United Kingdom and those from Germany and France; between imports from Germany and those from Italy; and between imports from France and those from the other-OECD category.

10. One annual observation is lost because of differencing. Note that the nine demand (supply) equations are not linearly dependent since they are expressed as quantity shares, as opposed to value shares. That is, the error terms do not add up to zero, and it is therefore not necessary to drop one equation.
11. The denominator of (15) can thus be interpreted as a fixed-weighted (i.e. direct) Laspeyres price index.
12. Note that these t values do not have the usual asymptotic properties given that the Hessian of the import cost function is of less than full rank; see Gourieroux, Holly, and Monfort (1982).
13. Estimates of Allen-Uzawa elasticities of substitution are not reported here for lack of space, but they can easily be calculated with the help of (6) using the estimates of Table 3 together with the import shares shown in Table 1.
14. Observed quantities were used to compute the partial price elasticities; this explains why the matrix of t values is symmetric.

We next turn to the export model. Parameter values are shown in Table 4, after the β_i's were set to 1980 shares for estimation purposes; 1987 estimates of the partial price elasticities of supply are shown in Table 5[15]. All own price elasticities are less than one. They are largest for exports to Germany and EFTA, and smallest for exports to Japan. The statistically most significant substitution effects are found to be between exports to Germany and exports to EFTA and Italy. Numerous other substitution relationships involve most prominently exports to North America on the one hand, and exports to Japan, Italy, and the Benelux on the other, as well as exports to Germany and those to Japan, France, and the small OECD nations. There is significant evidence of complementarity between exports to Canada and the United States, and exports to the United Kingdom, as well as exports to Japan and those to Italy. There are also some indications of complementarity (although not quite statistically significant) between exports to Germany and exports to North America and the Benelux; exports to Japan and those to EFTA and the Benelux; exports to EFTA and those to Italy; and exports to France and those to the small OECD nations.

Let us also consider the contribution of the passage of time on the composition of Swiss imports and exports. It is apparent from Tables 2 and 4 that the role of the time index is rather limited, except in the case of imports from Japan where the significantly positive estimate of γ_{2t} underscores the spectacular increase over the last three decades in the Japanese share of Swiss imports. It is noteworthy that neither γ_{1t} nor γ_{5t} are statistically significant: thus, relative price changes seem to be sufficient to explain the large drop in the North-American share of Swiss imports, and the substantial increase in the Germany's share. Similarly, on the export side, relative-price movements seem to adequately explain the large changes in the composition of Swiss exports which was evidenced by Table 1.

8. CONCLUSIONS

The main purpose of this paper was to model the geographical composition of Swiss foreign trade, and to assess the substitution possibilities allowed for by the Swiss import and export technologies. We found that significant substitution relationships do indeed exist, and that they can explain the changes in the geographical distribution of Switzerland's external trade. Generally speaking, substitution possibilities seem best between Switzerland's European trading partners, while complementarity relationships often implicate Switzerland's non-European partners, either amongst themselves, or with respect to some European nation. For instance, we have found that trade with Canada and the United States is a complement for trade with the United Kingdom, both on the import side and the export side.

15. Estimates of the elasticities of transformation can be calculated from the estimates of Table 5 with the help of (11), and using the 1987 export shares shown in Table 1.

A second purpose of this paper was to illustrate how the use of semiflexible functional forms makes it possible to estimate rather large demand and supply systems without constraining the sizes and the signs of the elasticities of substitution and transformation. Indeed, these elasticities are found to vary greatly between pairs of regions, and significant complementarity relationships have been uncovered both on the import side and on the export side. Such flexibility would not have been possible had we adopted the functional forms (CES and CET) generally used in the literature.

Table 1: Geographical Composition of Swiss Foreign Trade (percentage points)

	Imports		Exports	
	1958	1987	1958	1987
United States & Canada	15.93	6.35	16.01	12.35
Japan	1.01	5.08	1.88	4.82
EFTA	4.98	7.84	10.67	9.29
United Kingdom	6.50	6.74	7.42	9.38
West Germany	30.73	38.15	21.53	26.83
France	12.70	11.94	9.99	11.50
Italy	13.68	11.23	10.37	10.43
Benelux	10.67	8.23	10.04	6.54
Other OECD	3.80	4.44	12.09	8.86
Total OECD	100.00	100.00	100.00	100.00

Table 2: Import Aggregator Function
Symmetric Normalized Quadratic Semiflexible (K = 4) Function For
Parameter Estimates
(asymptotic-t values in parentheses)

τ_{11}	0.10765	(2.85)	τ_{21}	0.05791	(1.83)
τ_{31}	–0.06684	(–0.82)	τ_{41}	0.20531	(2.06)
τ_{51}	–0.12618	(–1.11)	τ_{61}	0.08496	(0.87)
τ_{71}	–0.11296	(–1.43)	τ_{81}	–0.11283	(–1.93)
τ_{22}	–0.06857	(–2.04)	τ_{32}	0.27785	(4.80)
τ_{42}	0.06310	(0.36)	τ_{52}	–0.22260	(–1.61)
τ_{62}	–0.04397	(–0.29)	τ_{72}	–0.09124	(–0.75)
τ_{82}	0.01453	(0.21)	τ_{33}	0.05812	(0.52)
τ_{43}	–0.07323	(–0.11)	τ_{53}	0.30511	(1.92)
τ_{63}	–0.06598	(–0.10)	τ_{73}	–0.29543	(–2.97)
τ_{83}	0.01660	(0.18)	τ_{44}	–0.30218	(–1.30)
τ_{54}	0.03694	(0.05)	τ_{64}	0.31756	(2.17)
τ_{74}	–0.02863	(–0.05)	τ_{84}	0.03254	(0.46)
γ_{1t}	–0.00260	(–1.49)	γ_{2t}	0.00140	(3.03)
γ_{3t}	0.00097	(1.10)	γ_{4t}	–0.00010	(–0.04)
γ_{5t}	0.00170	(0.81)	γ_{6t}	–0.00015	(–0.12)
γ_{7t}	–0.00073	(–0.90)	γ_{8t}	–0.00038	(–0.43)
γ_{9t}	0.00068	(1.64)	γ_{tt}	0.00001	(0.21)

Note:
The subscripts are defined as follows:
1. United States and Canada (M_1)
2. Japan (M_2)
3. EFTA (M_3)
4. United Kingdom (M_4)
5. West Germany (M_5)
6. France (M_6)
7. Italy (M_7)
8. Benelux (M_8)
9. Other OECD (M_9).

Table 3: Import Aggregator Function
Symmetric Normalized Quadratic Semiflexible Functional Form (K = 4)
1987 Partial Price Elasticity Estimates
(asymptotic-t values in parentheses)

$$\xi_{jk} \equiv \frac{\partial \ln\left[x_{M_j}\left(y_M, w_M\right)\right]}{\partial \ln\left(w_{M_k}\right)}$$

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9
ξ_{1k}	–0.193	–0.110	0.099	–0.334	0.239	–0.135	0.205	0.173	0.057
	(–1.45)	(–1.92)	(0.75)	(–2.56)	(1.21)	(–0.76)	(1.17)	(1.58)	(0.93)
ξ_{2k}	–0.137	–0.195	0.489	–0.154	–0.156	–0.158	0.014	0.151	0.148
	(–1.92)	(–1.84)	(2.85)	(–0.99)	(–0.90)	(–0.85)	(0.07)	(1.26)	(2.29)
ξ_{3k}	0.080	0.316	–1.046	–0.015	0.428	0.256	0.437	–0.147	–0.310
	(0.75)	(2.85)	(–2.95)	(–0.06)	(1.41)	(0.95)	(1.58)	(–0.68)	(–2.56)
ξ_{4k}	–0.315	–0.116	–0.017	–1.731	0.978	0.991	–0.032	0.379	–0.136
	(–2.56)	(–0.99)	(–0.06)	(–4.18)	(2.35)	(3.17)	(–0.11)	(1.91)	(–1.27)
ξ_{5k}	0.040	–0.021	0.088	0.173	–0.429	0.036	0.164	–0.038	–0.011
	(1.21)	(–0.90)	(1.41)	(2.35)	(–3.18)	(0.55)	(2.76)	(–0.74)	(–0.40)
ξ_{6k}	–0.072	–0.067	0.168	0.559	0.115	–0.904	–0.030	0.014	0.218
	(–0.76)	(–0.85)	(0.95)	(3.17)	(0.55)	(–3.02)	(–0.16)	(0.08)	(2.81)
ξ_{7k}	0.116	0.006	0.305	–0.019	0.556	–0.032	–1.034	–0.043	0.146
	(1.17)	(0.07)	(1.58)	(–0.11)	(2.76)	(–0.16)	(–3.85)	(–0.25)	(1.81)
ξ_{8k}	0.133	0.093	–0.140	0.310	–0.178	0.020	–0.059	–0.133	–0.047
	(1.58)	(1.26)	(–0.68)	(1.91)	(–0.74)	(0.08)	(–0.25)	(–1.12)	(–0.62)
ξ_{9k}	0.081	0.169	–0.548	–0.206	–0.097	0.585	0.370	–0.088	–0.267
	(0.93)	(2.29)	(–2.56)	(–1.27)	(–0.40)	(2.81)	(1.81)	(–0.62)	(–2.16)

Notes: These values are based on the estimates of Table 2; see the bottom of that table for a definition of the subscripts.

Table 4: Export Aggregator Function
Symmetric Normalized Quadratic Semiflexible (K = 4) Functional Form
Parameter Estimates
(asymptotic-t values in parentheses)

τ_{11}	–0.23932	(–4.24)	τ_{21}	0.12913	(2.47)
τ_{31}	0.07590	(0.88)	τ_{41}	–0.21691	(–2.70)
τ_{51}	–0.19470	(–1.62)	τ_{61}	0.15351	(2.14)
τ_{71}	0.14589	(1.56)	τ_{81}	0.12069	(1.88)
τ_{22}	–0.05683	(–1.43)	τ_{32}	–0.19009	(–1.45)
τ_{42}	0.03584	(0.23)	τ_{52}	0.05825	(0.22)
τ_{62}	0.14932	(1.62)	τ_{72}	–0.14004	(–1.02)
τ_{82}	–0.01270	(–0.11)	τ_{33}	0.13766	(1.17)
τ_{43}	0.09889	(0.99)	τ_{53}	–0.46520	(–6.70)
τ_{63}	0.07680	(0.74)	τ_{73}	0.13062	(0.97)
τ_{83}	–0.14588	(–2.85)	τ_{44}	$1.6\text{x}10^{-5}$	(0.00)
τ_{54}	$-2.7\text{x}10^{-5}$	(–0.00)	τ_{64}	$1.2\text{x}10^{-5}$	(0.00)
τ_{74}	$-3.6\text{x}10^{-6}$	(–0.00)	τ_{84}	$-1.4\text{x}10^{-5}$	(–0.00)
γ_{1t}	–0.00230	(–1.53)	δ_{2t}	0.00127	(1.03)
γ_{3t}	–0.00056	(–0.45)	δ_{4t}	0.00029	(0.24)
γ_{5t}	0.00182	(0.87)	δ_{6t}	0.00139	(1.67)
γ_{7t}	–0.00052	(–0.29)	δ_{8t}	–0.00110	(–1.49)
γ_{9t}	–0.00079	(–0.66)	δ_{tt}	–0.00006	(–1.73)

Note:
The subscripts are defined as follows:
1. United States and Canada (X_1)
2. Japan (X_2)
3. EFTA (X_3)
4. United Kingdom (X_4)
5. West Germany (X_5)
6. France (X_6)
7. Italy (X_7)
8. Benelux (X_8)
9. Other OECD (X_9).

Table 5: Export Aggregator Function
Symmetric Normalized Quadratic Semiflexible Functional Form (K = 4)
1987 Partial Price Elasticity Estimates
(asymptotic-t values in parentheses)

$$\Psi_{ih} \equiv \frac{\partial \ln\left[x_{X_i}(x_X, p_X)\right]}{\partial \ln\left(p_{X_h}\right)}$$

	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9
ψ_{1h}	0.499	–0.218	–0.166	0.422	0.378	–0.309	–0.302	–0.242	–0.062
	(2.12)	(–1.71)	(–0.89)	(2.58)	(1.39)	(–1.93)	(–1.53)	(–1.84)	(–0.32)
ψ_{2h}	–0.559	0.291	0.380	–0.508	–0.478	0.203	0.476	0.284	–0.091
	(–1.71)	(1.43)	(1.20)	(–1.88)	(–1.10)	(0.80)	(2.03)	(1.41)	(–0.33)
ψ_{3h}	–0.221	0.197	0.709	–0.113	–1.011	–0.056	0.632	–0.083	–0.051
	(–0.89)	(1.20)	(1.42)	(–0.33)	(–2.23)	(–0.20)	(1.83)	(–0.39)	(–0.15)
ψ_{4h}	0.555	–0.261	–0.112	0.583	–0.047	–0.213	–0.254	–0.422	0.170
	(2.58)	(–1.88)	(–0.33)	(1.35)	(–0.12)	(–0.79)	(–0.76)	(–1.86)	(0.55)
ψ_{5h}	0.174	–0.086	–0.350	–0.016	0.974	–0.206	–0.365	0.164	–0.289
	(1.39)	(–1.10)	(–2.23)	(–0.12)	(3.80)	(–1.78)	(–2.22)	(1.87)	(–1.77)
ψ_{6h}	–0.331	0.085	–0.046	–0.174	–0.479	0.432	0.107	0.053	0.352
	(–1.93)	(0.80)	(–0.20)	(–0.79)	(–1.78)	(1.73)	(0.48)	(0.34)	(1.73)
ψ_{7h}	–0.358	0.220	0.562	–0.228	–0.938	0.118	0.569	0.012	0.042
	(–1.53)	(2.03)	(1.83)	(–0.76)	(–2.22)	(0.48)	(1.42)	(0.07)	(0.14)
ψ_{8h}	–0.456	0.210	–0.119	–0.605	0.672	0.093	0.020	0.535	–0.350
	(–1.84)	(1.41)	(–0.39)	(–1.86)	(1.87)	(0.34)	(0.07)	(1.73)	(–1.11)
ψ_{9h}	–0.086	–0.049	–0.054	0.180	–0.875	0.457	0.049	–0.259	0.637
	(–0.32)	(–0.33)	(–0.15)	(0.55)	(–1.77)	(1.73)	(0.14)	(–1.11)	(1.35)

Notes: These values are based on the estimates of Table 4; see the bottom of that table for a definition of the subscripts.

REFERENCES

ARMINGTON, P.S. (1969). «A Theory of Demand for Products Distinguished by Place of Production», *IMF Staff Papers*, 16, pp 159–177.

ARROW, K.J., H.B. CHENERY, B.S. MINHAS, R.M. SOLOW (1961). «Capital-Labor Substitution and Economic Efficiency», *Review of Economics and Statistics*, 43, pp 225–250.

BERNDT, E.R. and N.E. SAVIN (1975). «Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbance», *Econometrica*, 43, pp 937–957.

DEATON, A. and J. MUELLBAUER (1980). *Economics and Consumer Behavior*, Cambridge: Cambridge University Press.

DIEWERT, W.E. (1974). «Applications of Duality Theory», in M.D. INTRILIGATOR and D.A. KENDRICK (eds.) *Frontiers of Quantitative Economics*, Vol. 2 Amsterdam: North-Holland.

DIEWERT, W.E. and T.J. WALES (1987). «Flexible Functional Forms and Global Curvature Conditions», *Econometrica*, 55, pp 43–68.

DIEWERT, W.E. and T.J. WALES (1988a). «Normalized Quadratic Systems of Consumer Demand Functions», *Journal of Business & Economic Statistics*, 6, pp 303–312.

DIEWERT, W.E. and T.J. WALES (1988b). «A Normalized Quadratic Semiflexible Functional Form», *Journal of Econometrics*, 37, pp 327–342.

DIEWERT, W.E. and T.J. WALES (1992). «Quadratic Spline Models for Producer's Supply and Demand Functions», *International Economic Review*, 33, pp 705–722.

GOURIEROUX, C., A. HOLLY, and A. MONFORT (1982). «Likelihood Ratio Test, Wald Test and Kuhn-Tucker Test in Linear Models with Inequality Constraints on the Regression Parameters», *Econometrica*, 50, pp 63–80.

HOTELLING, H. (1932). «Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions», *Journal of Political Economy*, 40, pp 577–616.

INTERNATIONAL MONETARY FUND, *Direction of Trade Statistics*, various issues.

INTERNATIONAL MONETARY FUND, *International Financial Statistics*, various issues.

KOHLI, U. (1978). «A Gross National Product Function and the Derived Demand for Imports and Supply of Exports», *Canadian Journal of Economics*, 11, pp 167–182.

KOHLI, U. (1985). «U.S. Imports by Origin: A System Approach», *Weltwirtschaftliches Archiv*, 121, pp 741–755.

KOHLI, U. (1991). *Technology, Duality and Foreign Trade: The GNP Function Approach to Modeling Imports and Exports*, London: Harvester Wheatsheaf.

KOHLI, U. (1992). «Production, Foreign Trade, and Global Curvature Conditions: Switzerland, 1948–1988», *Swiss Journal of Economics and Statistics*, 128, pp 3–20.

KOHLI, U. (1994). «Canadian Imports and Exports by Origin and Destination: A Semi-Flexible Approach», *Canadian Journal of Economics*, 27, pp 580–603.

KOHLI, U. and E.R. MOREY (1988). «U.S. Imports by Origin: A Characteristics Approach», *Kyklos*, 41, pp 51–74.

LAU, L.J. (1978). «Testing and Imposing Monotonicity, Convexity, and Quasiconvexity Constraints», in Melvyn Fuss and Daniel McFadden (eds.) *Production Economics: A Dual Approach to Theory and Applications*, Vol. 1 Amsterdam: North-Holland.

LEAMER, E.E. and R.M. STERN (1970). *Quantitative International Economics*, Boston, MA: Allyn and Bacon.

POWELL, A.A. and F.H.G. GRUEN (1968). «The Constant Elasticity of Transformation Production Frontier and Linear Supply System», *International Economic Review*, 9, pp 315–328.

SHEPHARD, R.W. (1953). *Cost and Production Functions* Princeton, NJ: Princeton University Press.

UZAWA, H. (1962). «Production Functions with Constant Elasticities of Substitution», *Review of Economic Studies*, 23, pp 101–108.

WHITE, K.J. (1978). «A General Computer Program for Econometric Methods – SHAZAM», *Econometrica*, 46, pp 239–240.

WILEY, D.E., W.H. SCHMIDT, and W.J. BRAMBLE (1973). «Studies of a Class of Covariance Structure Models», *Journal of the American Statistical Association*, 68, pp 317–323.

WOODLAND, A.D. (1982). *International Trade and Resource Allocation*, Amsterdam: North-Holland.

SUMMARY

This paper models the geographical composition of Swiss foreign trade. Imports and exports are disaggregated into nine regions which together cover the entire OECD area. Given the large size of the model, the use of a flexible functional form is problematic. An attractive alternative is provided by semiflexible functional forms. These allow for large savings in terms of the number of parameters, and yet they are still able, under certain conditions, to provide a second-order approximation to an arbitrary aggregator function. Our results show that there are significant substitution – but also complementarity – relationships in Swiss foreign trade.

ZUSAMMENFASSUNG

Dieser Aufsatz modelliert die geographische Zusammensetzung des schweizerischen Aussenhandelsgebiets. Importe und Exporte werden in neun Regionen unterteilt, die die gesamte OECD-Fläche abdecken. Der Gebrauch einer flexiblen funktionalen Form ist wegen der beträchtlichen Grösse des Modells problematisch. Eine attraktive Alternative bilden semiflexible funktionale Formen. Diese erlauben eine substantielle Reduktion in der Anzahl Parameter. Unter gewissen Bedingungen ermöglichen sie zudem eine Annäherung zweiten Grades an eine arbiträre aggregierte Funktion. Die Resultate zeigen, dass es bedeutsame substitutive – aber auch komplementäre – Beziehungen im schweizerischen Aussenhandel gibt.

RESUME

Dans cette étude nous modélisons la composition des importations et exportations suisses par région d'origine et de destination. Tant les importations que les exportations sont identifiées selon neuf régions qui ensemble recouvrent la totalité de la zone de l'OCDE. Etant donné la large taille du modèle, l'utilisation d'une forme flexible est peu aisée; une alternative attrayante est l'utilisation d'une forme semi-flexible. Les formes semi-flexibles permettent une économie importante en terme de paramètres, tout en restant capables, sous certaines conditions, de fournir une approximation du second ordre d'une fonction arbitraire. Nos résultats montrent qu'il y a d'importants effets de substitution – mais aussi de complémentarité – dans le commerce extérieur de la Suisse.