

Extracting Market Expectations from Option Prices: Two Case Studies in Market Perceptions of the ECB's Monetary Policy 1999/2000

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1. INTRODUCTION

It is long understood that prices of financial assets contain information on how market participants expect asset prices to move in the future. For example, the forward price of an asset represents the price a risk-neutral investor expects to prevail at the time the contract expires. Due to their forward-looking nature, asset prices instantly respond to new information and therefore are a natural indicator of changes in market participants' expectations.

The rapidly increasing liquidity of financial derivatives markets led researchers to attempt to gain further insights into the evolution of expectations on financial markets by using information embodied in option prices. For example, by inverting the Black-Scholes option valuation formula using observed option prices one can estimate the "implied volatility" of the underlying asset which represents the expected average volatility of the underlying until the option's expiration (e.g. CAMPA and CHANG, 1995; DUMAS et. al., 1998; LAMOUREUX and LASTRAPES, 1993; SCOTT, 1992).¹

In recent years a number of more ambitious techniques for uncovering information on market expectations implicit in option prices has been developed. These techniques allow for the extraction of the whole risk-neutral probability density function with re-

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1. There are many papers that study the forecasting ability of implied volatility for future standard deviations of the underlying asset's return. See, for example BANK OF JAPAN (1995), CANINA and FIGLEWSKI (1993), DAY and LEWIS (1992), DUMAS et. al. (1998), FLEMING (1998), JORION (1995) or NEUHAUS (1995).

spect to the price of the underlying asset on a specific future date. Thus, they provide superior information on future asset prices as expected by market participants than just a futures/forward price or implied volatility. These techniques have been applied to various problems that range from exchange rate expectations (CAMPA, CHANG, and REIDER, 1998; COOPER and TALBOT, 1999) to the credibility of exchange rate regimes (e.g. CAMPA and CHANG, 1996, 1998; CAMPA, CHANG, and REFALO, 1998; MALZ, 1996), to commodity markets (e.g. MELICK and THOMAS, 1997) and interest rate expectations (e.g. ABKEN, MADAN, and RAMAMURTIE, 1996; COUTANT, JONDEAU, and ROCKINGER, 1998; SÖDERLIND, 2000). Moreover, central banks have taken considerable interest in the development and application of these techniques.²

In particular, an insight into changes in market expectations around important events related to monetary policy actions is thought to provide information that might be useful for assessing central bank credibility and the effectiveness of monetary policy as perceived by financial markets (BAHRA, 1997). Furthermore, these techniques could be helpful in providing information on how effective the central bank was in communicating its monetary policy intentions and in stabilizing expectations with respect to monetary policy. For example, if a monetary policy action is perfectly anticipated, the probability density function describing the market's assessment of the likelihood of various future outcomes for an interest rate should remain unchanged. These aspects are particularly relevant for the newly established ECB as those techniques can be applied to yield an insight into how market participants perceive the ECB's monetary policy.

This paper is intended to examine the capability of these new techniques to capture the evolution of market expectations. For this purpose we will make use of one of these new techniques in order to extract market expectations concerning short-term interest rates and to track the evolution of market expectations around the time of recent important monetary policy decisions. The plausibility of the results will be examined by considering how the results conform to market comment prevailing at that time. More specifically, we will fit a mixture of two lognormal densities to observed prices of LIFFE options on three month Euribor futures contracts.³ The first estimation period starts on 25 October, 1999 and ends 12 November. The second goes from 17 January, 2000 to 11 February. These periods cover the decisions of the ECB-council to raise interest rates on 4 November, 1999 and 3 February, 2000, as well as the decline of the Euro exchange rate below dollar parity at the end of January, 2000.

The next section briefly summarizes the theoretical foundations that relate risk-neutral probability densities to option prices. Section three presents the estimation techni-

2. See for example BAHRA (1997), COOPER and TALBOT (1999), COUTANT, JONDEAU and ROCKINGER (1998), FORNARI and VIOLI (1998), GALATI and MELICK (1999), LEVIN, McMANUS and WATT (1998), MALZ (1996), McMANUS (1999), MELICK and THOMAS (1997), NAKAMURA and SHIRATSUKA (1999), NEUHAUS (1995), and BANK FOR INTERNATIONAL SETTLEMENTS (1999).
3. Examples for this approach are BAHRA (1997), JONDEAU and ROCKINGER (2000), LEAHY and THOMAS (1996), MELICK and THOMAS (1997, 1998), SÖDERLIND (2000), and SÖDERLIND and SVENSSON (1997).

que, whereas section four contains some information on the data set. In section five the results are presented and discussed. Finally, section six summarizes and provides some concluding remarks.

2. OPTION PRICES AS EXPECTED VALUES

As shown in Cox and Ross (1976) the price of a European option on an underlying asset S can be expressed as the discounted expected value of the option payoff at expiration,⁴

$$v(t, S(t)) = e^{-r(T-t)} E[h(S(T))|S(t)], \quad (1)$$

where T is the expiration date, r is the riskless rate of interest and $h(S(T))$ is the option payoff function at expiration. For a European call option with strike price K the payoff function is $h_c(S(T)) = \max[S(T) - K, 0]$, and for a European put option $h_p(S(T)) = \max[K - S(T), 0]$. Consequently, for the call,

$$\begin{aligned} c(t, S(t), K) &= e^{-r(T-t)} E[\max[S(T) - K, 0]|S(t)] \\ &= e^{-r(T-t)} \int_K^{\infty} (S(T) - K) f(S(T)|S(t)) dS(T) \end{aligned} \quad (2)$$

and for the put,

$$\begin{aligned} p(t, S(t), K) &= e^{-r(T-t)} E[\max[K - S(T), 0]|S(t)] \\ &= e^{-r(T-t)} \int_0^K (K - S(T)) f(S(T)|S(t)) dS(T) \end{aligned} \quad (3)$$

where the expectation is taken with respect to the probability density function (PDF) $f(S(T))$ conditional on $S(t)$.⁵ $f(S(T))$ is called the *risk-neutral* probability density function.⁶ It differs from the conditional density derived from the true stochastic process of the underlying asset because of the market participants' risk preferences.

4. The owner of an European call option has the right but not the obligation to buy the underlying asset at a prespecified price (strike price or exercise price) on a given date (expiration date), whereas a European put option provides him with the right but not the obligation to sell the underlying asset at a prespecified price on the expiration date.
5. For simplicity $f(S(T)|S(t))$ will be written as $f(S(T))$.
6. This density is also called "state price deflator" (e.g. DUFFIE, 1996) or "state price density" (e.g. AÏT-SAHALIA and LO (1998). In the asset-pricing literature it is the "equivalent martingale measure" (HARRISON and KREPS, 1979) or "pricing measure". As long as markets are complete the three concepts coincide. See CONT (1999) for a survey.

3. ESTIMATION TECHNIQUE

Because of the relationship stated in (2) and (3) it is possible to infer the risk-neutral PDF from observed prices of European options. Many different techniques have been applied to option prices in order to extract the implied risk-neutral PDF. Along broad lines, these techniques can be classified into two categories. One group directly utilizes equations (2) and (3) by making assumptions about the functional form of the PDF and fitting the resulting theoretical option prices to observed option prices. The other techniques use the result by BREEDEN and LITZENBERGER (1978) who showed that the second derivative of the call-pricing function with respect to the exercise price equals the discounted risk-neutral density function. This allows to obtain the risk-neutral density by constructing a continuous call-pricing function $c(K)$ and differentiating twice numerically.⁷

In the first class, a sufficiently flexible parametric distribution is assumed to be capable of representing the implied risk-neutral PDF. Then parameter estimates are obtained by fitting the chosen distribution to observed option prices via equations (2) and (3). Assuming a parametric density for the implied risk-neutral PDF corresponds to imposing some structure on the terminal distribution of the price of the underlying asset. However, this terminal distribution can be consistent with many stochastic processes of the asset price (MELICK and THOMAS, 1997) and does not impose a particular law of motion on the underlying like, for example, exponential Brownian motion. One parameterization that proved to be particularly popular is a mixture of lognormal densities.⁸

RITCHEY (1990) has shown that deviations of the conditional price distribution of the underlying asset from the Black-Scholes (1973)-assumption of a lognormal distribution (i.e. the price process being exponential Brownian motion) can be accommodated within an option valuation model which assumes the conditional price distribution to be a mixture of lognormal distributions.

Building on this result MELICK and THOMAS (1997) and BAHRA (1997) assume the risk-neutral PDF in (2) and (3) to be a mixture of k lognormal PDFs,

$$f(S(T)|S(t)) = \sum_{i=1}^k [\theta_i L(\alpha_i, \beta_i; S(T))] \quad (4)$$

where $L(\alpha_i, \beta_i, S(T))$ is the lognormal PDF with mean α_i and standard deviation β_i ,

$$L_i(\alpha_i, \beta_i; S(T)) = \frac{1}{S(T)\beta_i\sqrt{2\pi}} e^{[-(\ln S(T)-\alpha_i)^2]/2\beta_i^2} \quad (5)$$

7. This classification is from CHANG and MELICK (1999). See JACKWERTH (1999) for a survey on the various techniques.
8. See, for example, MELICK and THOMAS (1995, 1998), SÖDERLIND (2000), SÖDERLIND and SVENSSON (1997).

and weights θ_i which sum to one,

$$\sum_{i=1}^k \theta_i = 1. \quad (6)$$

The parameters of the component lognormal distributions can be time dependent. For example, in the special case of Black's model for pricing futures options (BLACK, 1976) which implies only one lognormal distribution we have $\theta_1 = 1$, $\alpha_1 = \log(S(t)) - \frac{1}{2}\sigma^2(T-t)$ and $\beta_1 = \sigma\sqrt{T-t}$, where σ is the (annualized) volatility of the future contract and $S(t) = F(t)$ the current futures price.

The number k of lognormal distributions employed to model the risk-neutral PDF depends on the quality and quantity of available option prices. Since the number of traded options – in our case futures options – which are sufficiently liquid to be used for estimation is usually quite low and an increase in k by just one raises the number of parameters to be estimated by three (mean, standard deviation and associated weighting factor), most authors decide to use $k = 2$ or $k = 3$.⁹

Following McMANUS (1999) and BAHRA (1997) we choose $k = 2$, which leads to the following expressions for equations (2) and (3),¹⁰

$$c(t, K) = e^{-r(T-t)} \int_K^\infty (S(T) - K) [\theta L(\alpha_1, \beta_1; S_T) + (1 - \theta) L(\alpha_2, \beta_2; S_T)] dS(T), \quad (7)$$

$$p(t, K) = e^{-r(T-t)} \int_0^K (K - S(T)) [\theta L(\alpha_1, \beta_1; S_T) + (1 - \theta) L(\alpha_2, \beta_2; S_T)] dS(T). \quad (8)$$

Equations (7) and (8) yield, for any admissible parameter vector $\eta' = (\alpha_1, \alpha_2, \beta_1, \beta_2, \theta)$, theoretical option prices $\hat{c}(t, \eta, K)$ and $\hat{p}(t, \eta, K)$.

Given observed option prices and the riskless rate of interest the parameters are estimated by minimizing the sum of squared deviations between theoretical and observed option prices,

$$\begin{aligned} & \min_{\eta} \sum_{i=1}^n [c(t, S(t), K) - \hat{c}(t, \eta, K)]^2 \\ & + \sum_{i=1}^m [p(t, S(t), K) - \hat{p}(t, \eta, K)]^2 + [\hat{F}(t, \eta) - F(t)]^2, \end{aligned} \quad (9)$$

9. Three lognormal distributions are used in MELICK and THOMAS (1997), whereas LEVIN, McMANUS and WATT (1998), McMANUS (1999), SÖDERLIND (2000), SÖDERLIND and SVENSSON (1997) among others, opt for just two lognormal PDFs.

10. The dependence of the option prices on $S(t)$ is no longer shown explicitly but has been incorporated into the distributional parameters.

subject to $\beta_1 > 0, \beta_2 > 0, 0 \leq \theta \leq 1$. The last term is the squared deviation of the theoretical futures price $\hat{F}(t, \eta, S(t))$ from the actual futures price $F(t, S(t))$ of the underlying asset. The theoretical futures price is the expected value of the underlying at expiration where expectations are taken with respect to the risk-neutral PDF,¹¹

$$\hat{F}(t, \eta) = \theta e^{\alpha_1 + \frac{1}{2}\beta_1^2} + (1 - \theta) e^{\alpha_2 + \frac{1}{2}\beta_2^2}. \quad (10)$$

BAHRA (1997) shows that the call and put valuation formulae (7) and (8) can be written in closed forms as,

$$c(t, K) = \quad (11)$$

$$e^{-r(T-t)} \left\{ \theta \left[e^{\alpha_1 + \frac{1}{2}\beta_1^2} N(d_1) - K \cdot N(d_2) \right] + (1 - \theta) \left[e^{\alpha_2 + \frac{1}{2}\beta_2^2} N(d_3) - K \cdot N(d_4) \right] \right\},$$

$$p(t, K) = \quad (12)$$

$$e^{-r(T-t)} \left\{ \theta \left[-e^{\alpha_1 + \frac{1}{2}\beta_1^2} N(-d_1) + K \cdot N(-d_2) \right] + (1 - \theta) \left[e^{\alpha_2 + \frac{1}{2}\beta_2^2} N(-d_3) + K \cdot N(-d_4) \right] \right\}.$$

$N(\bullet)$ is the cumulative standard normal distribution and,

$$d_1 = \frac{-\ln K + \alpha_1 + \beta_1^2}{\beta_1}, \quad d_2 = d_1 - \beta_1, \quad (13)$$

$$d_3 = \frac{-\ln K + \alpha_2 + \beta_2^2}{\beta_2}, \quad d_4 = d_3 - \beta_2. \quad (14)$$

The minimization problem (9) can be simplified further by considering the institutional features of LIFFE short-term interest rate (STIR) options examined in this paper: The underlying asset of LIFFE STIR options is a futures contract on a short-term interest rate with price $F(t, r_{\text{eur}}(t))$. Furthermore, these options are subject to a futures-style margining procedure.¹²

These characteristics lead to two simplifications: First, no discount factor appears in (10) because the price of a futures contract in a risk neutral world is a martingale. Second, as shown in SCOTT and CHEN (1993) the discount factors $e^{-r(T-t)}$ in (11) and (12) disappear.

In equations (11) and (12) the underlying's price and the exercise price are expressed in terms of the futures contract. However, by noting that the underlying futures contract

11. The expected value of a lognormal distributed variable $x \sim L(\mu, \sigma)$ is $E(x) = \exp(\mu + \frac{1}{2}\sigma^2)$.

12. Usually the buyer has to pay the option premium up front. However, at LIFFE he has to deposit an "initial margin" as collateral and gains and losses on his position are added to or subtracted from his margin account. Because the money in the margin account remains the property of the buyer. He avoids paying the option premium in advance. For a description of LIFFE STIR options see LIFFE (1999).

is quoted as one hundred minus the interest rate r_{eur} so that the interest rate implied by the futures price is given as $[100 - F(t, r_{\text{eur}}(t))]$, the expressions can be reformulated in terms of the interest rate r_{eur} .

The structure offered by (11) and (12) is sufficiently flexible to accommodate a variety of shapes of the resulting risk-neutral PDF including the possibility of bimodal distributions. Furthermore, the mixture of lognormal PDFs implies smooth behaviour of the tail probabilities which decline monotonically and quickly enough not to exert an unreasonable influence on the kurtosis of the distribution (CAMPA, CHANG and REIDER, 1998).

The parameter vector η is estimated via nonlinear optimization. It turned out that the optimization procedure applied encountered some difficulties in converging to the global minimum.¹³ Therefore a grid search was performed over a set of 5400 possible starting values for the optimization procedure. Furthermore, the weighting factor was restricted at $0.05 < \theta < 0.95$ to avoid the parameters of one of the mixture distributions to become unidentified when θ is close to zero or one, respectively.

4. DATA

This study uses two sets of option prices from the London International Financial Futures and Options Exchange (LIFFE). In both cases, the underlying asset is a three month Euribor interest rate futures contract, i.e. a futures contract on a short-term interbank interest rate.

The options on short-term interest rate futures traded at LIFFE are American options, which can be exercised at any time up to the expiration date. However, the futures-style margining procedure applied to STIR options at LIFFE leads to the result that the American options are actually priced as European options. Early exercise, i.e. exercise before the expiration date will not take place because there is no opportunity cost of holding a long position in options.¹⁴

We will use settlement prices of options and the underlying futures contract, which are each day determined by the exchange at the close of trading in the respective contract. The use of settlement prices can be justified on theoretical grounds by noting that during trading time trades based on liquidity considerations may temporarily move prices away from their equilibrium values, a problem which should be mitigated by using settlement prices (MELICK and THOMAS, 1998). Furthermore, since the members of the exchange also act as brokers or hold positions in options themselves there are sufficient incentives to determine settlement prices close to their equilibrium values (SÖDERLIND, 2000).

13. Compare COUTANT, JONDEAU, and ROCKINGER (1998). The optimization was carried out with the Broyden-Fletcher-Goldfarb-Shanno algorithm.

14. See SCOTT and CHEN (1993) for a formal argument.

The first subset of data consists of option and futures prices on 15 trading days from 25 October, 1999 to 12 November, 1999 bracketing the decision by the ECB to raise the interest rates on its main refinancing operations and on the marginal lending and borrowing facilities on 4 November 1999. The selected futures contract was the one on the regular cycle expiring in March 2000 with the options expiring at the same date.¹⁵ The March contract was chosen to give an insight into the evolution of interest rate expectations sufficiently far into the future and to avoid problems associated with choosing a contract with time to expiration of less than one month.¹⁶

The other time window considered here starts on 17 January, 2000, shortly before the exchange rate of the Euro vs. the US-Dollar first dropped below parity on 24 January, 2000. It ends on 11 February, 2000, thus extending 20 trading days including the interest rate increase by the ECB (3 February, 2000). For this period options and the futures contract with expiration in June were selected.

By selecting options with expiration near the futures contract's expiration date the implied risk-neutral PDF can be interpreted as the density function of the actual three month Euribor interest rate prevailing in March and June 2000, respectively.

One can question the justification of including in our data set more than one trading day after the event of interest took place. If market participants form rational expectations and if new information is incorporated into prices sufficiently quickly all the effects of the event considered will be visible in the change of the risk-neutral PDF from one day to the other. Unfortunately, as shown later, there are considerable changes in the shape of the risk-neutral PDF as well as in moments and percentiles derived from it, even under "normal" circumstances. Therefore it is very difficult to separate the effects of, say, monetary policy actions on the risk-neutral PDF from the ordinary "noise". As a remedy we try to look for changes in the risk-neutral PDFs that exhibit some persistence. We therefore extend the data set to the week after the event took place. In deciding on how many additional trading days to include we face a trade-off: If we add too few observations, we will not be able to "average away" the normal changes in the risk-neutral PDF. On the other hand, if we add too many trading days to the sample, the effects we wish to study will be blurred by the additional information the market participants learn during that period.

By choosing to add a further week of observations to our sample we might have struck the balance too far on the latter side of the trade-off. However, during the trading days added to our sample there were no other important events related to monetary policy apart from those investigated. We can therefore assume that new information feeding into the risk-neutral PDF was not biased in favour of showing expectations of

15. The regular cycle of expiry months offered by LIFFE is March, June, September and December. There are also traded options expiring in the three nearest calendar months which, however, lack the liquidity of the options on the regular expiration cycle.
16. Because of the low volatility of short-term-interest rates the range of strike prices that are actively traded will decrease dramatically when the expiration date is approached thereby reducing the degrees of freedom in the estimation.

increasing or decreasing interest rates. Consequently, these “neutral” observations might induce a bias towards rejecting any significant effects of the events to be considered.¹⁷

Options on the three month Euribor futures contract are available at strike prices spaced at 25 basis points. An option is on one futures contract of 1,000,000 Euros. The minimum tick size, i.e the minimum price movement of the options is 0.005 which corresponds to 12.50 Euros (LIFFE, 1999).

For each day, we select pairs of calls and puts (i.e. n is set equal to m in (9) as, for example, in BAHRA, 1997) with respect to the following criteria: First, all pairs of options which contain one option with zero price are eliminated. In these pairs one option is far out-of-the-money and one option is far in-the-money, both types not traded very actively and thus containing few reliable information on market participants’ expectations.¹⁸ Second, we drop from the remaining sample all pairs of options which contain one option with price equal to the minimum tick size of 0.005. Since option prices are discretely spaced at 0.005, options with prices that low are suspect to carry large measurement errors into the sample.

This selection leaves a total of 187 pairs of option prices for the period in October/November 1999 and 224 pairs for the period in January/February 2000.

5. RESULTS

For each day in the sample the parameter vector $\eta' = (\alpha_1, \alpha_2, \beta_1, \beta_2, \theta)$ was estimated using (9) with (11) and (12). The estimated parameter values were then plugged into (4) and (5) to construct the implied risk-neutral PDF for that day. Unfortunately, on two occasions (24 January and 4 February) the estimated standard deviation of one of the distributions collapsed towards zero, thus rendering the numerical computation of the implied risk-neutral PDF impossible. For this reason, those two days were eliminated from the sample for the subsequent examinations. Furthermore the algorithm showed some problems to converge for observations from 27 January.

17. Another argument to consider is that extending the data set by another week of observations relative to the event to be studied can mitigate false conclusions stemming from day-of-the-week effects.

18. Including for example a put option where the corresponding call has zero price implies a risk-neutral probability of zero at the respective strike, because the second derivative of the call price with respect to strike becomes zero. Because of put-call-parity and since both options are priced on the same distribution the put option also implies a risk-neutral probability of zero. Choosing only pairs of calls and puts therefore does include the domain where risk-neutral probabilities are positive and larger than zero.

5.1. *Inspection of densities*

Before giving an extensive account of results relating to various statistics computed from the estimated PDFs it will be useful to establish some of their basic features by inspection of the densities' shapes. Figure 1 shows selected implied risk-neutral PDFs. Significant changes in their shapes and location should provide evidence on the evolution of market participants' expectations.

Figure 1: Implied probability density and cumulative distribution functions

Figure 1a: Three month Euribor futures options (March 2000)

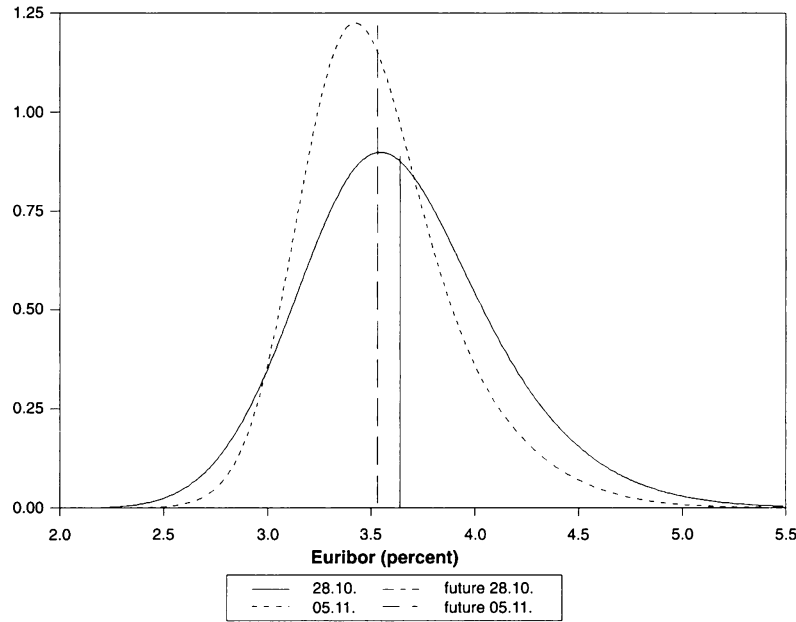


Figure 1b: Three month Euribor futures options (June 2000)

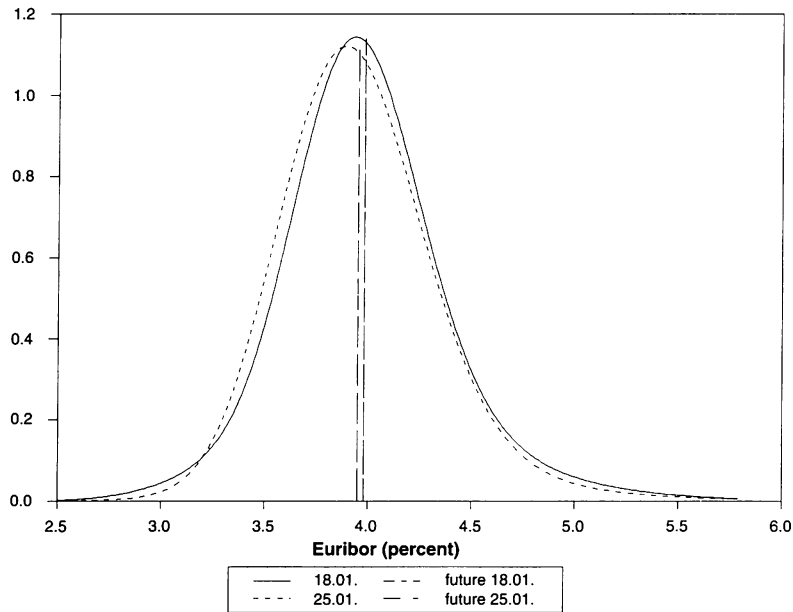


Figure 1c: Three month Euribor futures options (June 2000)

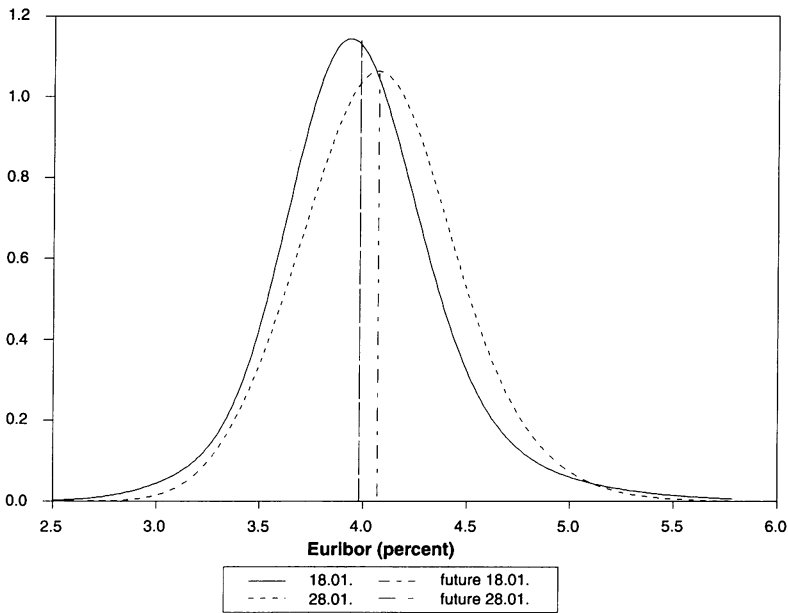


Figure 1d: Three month Euribor futures options (June 2000)

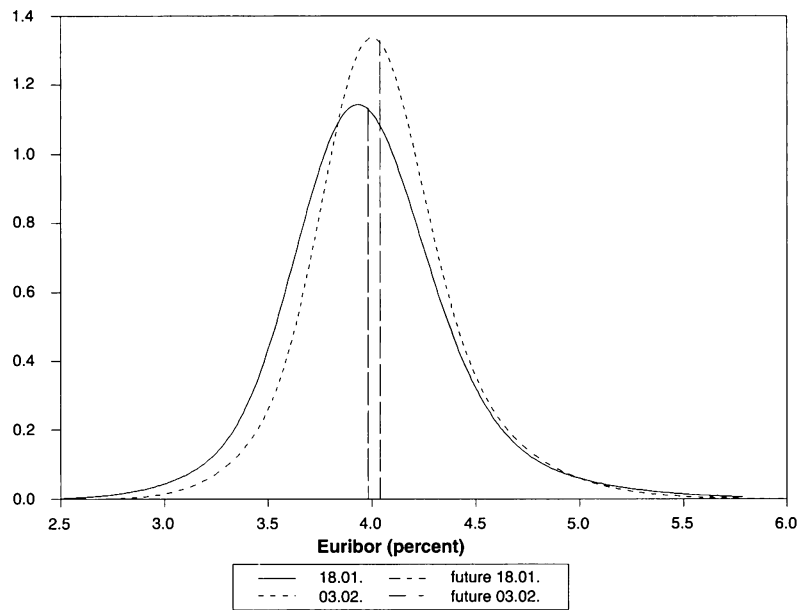
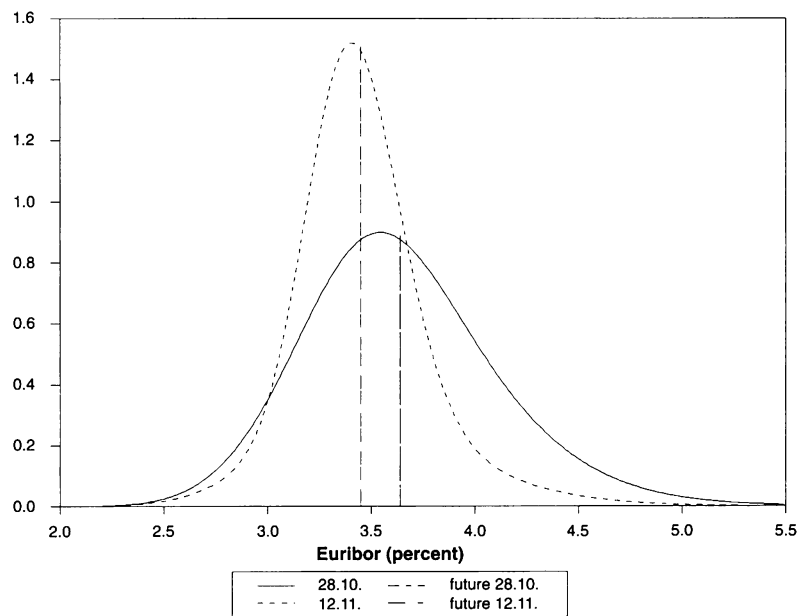


Figure 1e: Three month Euribor futures options (March 2000)



In the first figure the estimated risk-neutral densities of the three month Euribor rate in March 2000 are shown together with the futures implied interest rate ($100 - F$) on 28 October and 5 November, 1999, after the ECB-council's decision to raise interest rates on 4 November. Both PDFs are representative for the shapes of PDF before and after the council meeting. The mean (the futures implied interest rate) slightly shifted to the left, implying a lower expected interest rate and the distribution contracted around the mean showing a lower dispersion of the expected interest rate. Market commentary stated that the ECB-council's decision to raise interest rates was widely anticipated more than one week in advance.¹⁹ This impression is supported by the estimated PDFs because there is only a small change in the distributions' mean. Nevertheless, there also are some indications that that on the market there was still some uncertainty around concerning the upcoming meeting of the ECB-council because the distribution's dispersion seems to be slightly lower after the council meeting.

The second row contains the PDFs estimated from options and the future contract expiring in June 2000 as estimated for 18 January, 2000, some days before the Euro first dropped below dollar-parity on 24 January. The second PDF, which is from 25 January is positioned slightly to the left of the former one but does not exhibit a significantly different shape. Therefore these PDFs do not indicate that the continuing decline of the Euro exchange rate made market participants to revise their expectations of future short-term interest rates.

How did the situation change four days later? In the third row the respective distributions are plotted for 18 January and 28 January one day after the Euro reference value dropped below 1 US-Dollar. Furthermore, on January 28, the Euro reference value declined by 1.28 cents against the dollar and commentators expressed the opinion that the ECB-council, when meeting the following Thursday, would decide to raise interest rates once again.²⁰ These comments are clearly consistent with the picture provided by the estimated distributions. The futures implied interest rate increased and the PDF moved to the right. On the first glance, it does not appear that market uncertainty increased in the wake of these events because there seems not to be a marked increase in the dispersion of the density.

Finally, the distributions estimated for 28 January and 3 February, the day the ECB-council decided to raise interest rates again by 25 basis points, are compared.²¹ The mean of the distribution exhibits only a very small change (three basis points) whereas

19. For example, "In Euroland an increase in the base interest rate is deemed to be very likely but already seems to have been incorporated into (asset) prices" (Handelsblatt, 01 September 1999, p. 37) or, "For most market participants it is clear: The European Central Bank will increase the base interest rate from 2.5 % to 3 % on the next Thursday" (Handelsblatt, 03 November 1999, p. 37). (Translations by the author).

20. "Due to the weakness of the Euro traders are confident that the ECB will raise the base interest rates for the Euro area at the next council meeting in the following week." (Handelsblatt, 28 January 2000, p. 1, Translation by the author).

21. The data consist of settlement prices which are established at the close of trading at 4.10 p.m.

the distributional shape becomes more concentrated around the futures price and dispersion decreases. In combination with the evidence presented in the preceding paragraph the only very small change in the estimated mean indicates that the ECB's decision appears to have been anticipated and that market participants began to expect the ECB to react to the decline in the Euro by raising interest rates after the Euro dropped below parity.²² Since the expectation's dispersion declines, market participants' expectations appear to have retained substantial uncertainty relating to the fact that the ECB had denied to consider taking any actions solely based on the evolution of the Euro exchange rate and that this uncertainty was resolved by the ECB's decision.

Altogether, these results can be deemed consistent with accounts of prevailing market sentiments. Particularly, it has been shown that both interest rate increases were expected by market participants. However, it remains to question the significance of the observed changes in the implied distributions. Due to the limited number of observed option prices the estimated distributions often changed their shapes considerably from day to day. It remains to be seen if the changes observed above are more than just "normal" fluctuations.

5.2. Implied moments

Using the estimated densities higher moments of the implied risk-neutral PDF were computed and studied over time.²³ The third and fourth central moment were calculated to provide information on skewness and kurtosis of the estimated PDF. The skewness and kurtosis measures presented here refer to the *logarithm* of the expected interest rate and therefore can be compared to those of the normal distribution.²⁴ Figure 2 shows the time series of skewness and excess kurtosis (i.e. kurtosis – 3). Both subsamples exhibit considerable day-to-day variability in the moments series.

22. "In face of the ongoing weakness of the Euro more and more market participants consider a change in interest rates by the ECB on Thursday... to be very likely." (Handelsblatt, 01 February 2000, p. 1, Translation by the author).

23. In interpreting the relevance of observed changes in moments of higher order of the PDF it should be kept in mind that the changes in the estimated PDF cannot be considered independently from the changes in the futures implied interest rate – the mean of the PDF. It is argued that on a day-to-day basis changes in the futures implied interest rate exert the most dominant influence on the distribution. See LEVIN, McMANUS and WATT (1998).

24. Denote with cm_3 and cm_4 the third and fourth central moments of the PDF. The calculated measures were: $skew = cm_3/s^3$ and $kurtosis = cm_4/s^4$, where s is the standard deviation of the estimated PDF. For the normal distribution we have skew = 0 and kurtosis = 3.

Figure 2: Skewness and excess kurtosis of implied PDF

Figure 2a: Skewness and Kurtosis implicit in three month Euribor futures options
25. 10. –12. 11. 99

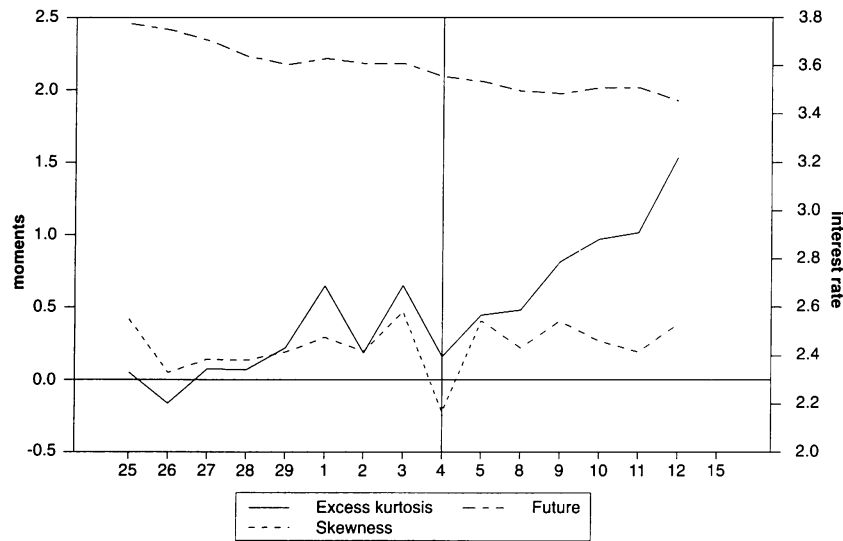
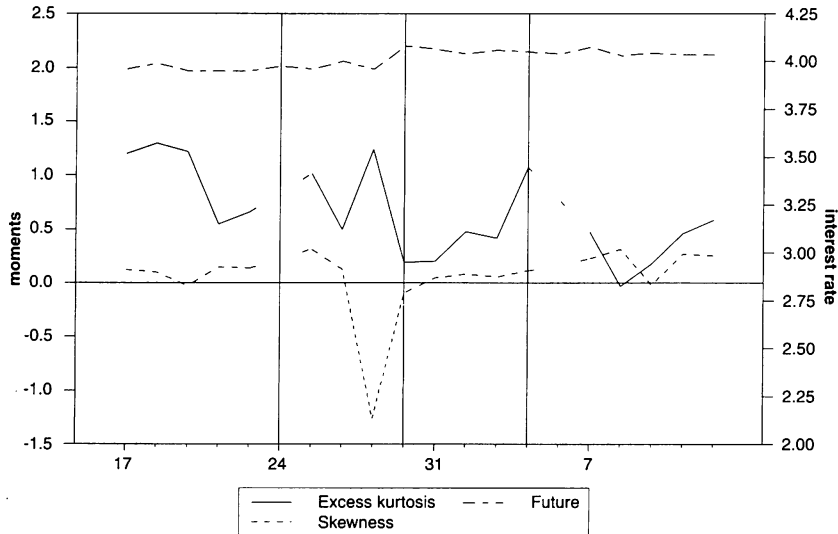


Figure 2b: Skewness and Kurtosis implicit in three month Euribor futures options
17. 01. –11. 02. 00



For the first subsample (October/November 1999) the risk-neutral PDF is positively skewed apart from 4 November, implying a higher probability of large interest rate increases than of large decreases.²⁵ Skewness becomes negative on the day of the interest rate increase but immediately returns to positive values again. The average skewness is 0.235. Excess kurtosis is positive for 14 out of 15 days with an average value of 0.478. In both subsamples, excess kurtosis appears to exhibit a time trend.²⁶

It is difficult to interpret the strong positive trend in excess kurtosis observed over the first subsample. An increase in a distribution's excess kurtosis implies fatter tails and/or a more concentrated, i.e. sharper center. That means the probability of both only very small movements around from the center and very large up and down movements increases. The first interpretation would suggest that uncertainty decreases while the second one would point to increasing uncertainty.²⁷ Looking at the last panel in figure 1 which shows risk-neutral PDFs for 28 October and 12 November – the day with the largest excess kurtosis – the first interpretation, i.e. a general decrease in uncertainty about the future interest rate which was taking place over some weeks seems to be the most appropriate.

The presence of a time trend might obscure the effect of the ECB-council's decision on excess kurtosis. Therefore we regressed excess kurtosis on a time trend and a dummy variable set to one from 4 November until the end of the subsample.²⁸ The results are presented in table 1. The dummy variable is significant at the 5 % level and the coefficient estimate is negative. This decrease in excess kurtosis after the council meeting could again be interpreted in two possible ways as explained above. However, in section 5.3 it is shown that a scaled interpercentile range, a measure of uncertainty which is based on the extreme tails of the PDF decreases significantly after the council meeting. This implies that the decrease in *time adjusted* excess kurtosis after 4 November is mostly due to a flattening of the distribution's tails and therefore indicates a decrease in uncertainty.

We performed a similar regression for the skewness measure without any significant results. Since skewness represents a directional bias in expectations we would not expect it to be significantly affected by the ECB-dummy if the policy action was anticipated.

25. A positively skewed distribution has the median to the left of the mean. That implies that an interest rate below the current futures rate is seen by market participants to have a likelihood of more than 50 %.
26. As for the implied percentile differences, there can be maturity dependence in implied moments. For example, if the true stochastic process of the underlying asset is a random walk, excess kurtosis and skewness of the true conditional density would be $(1/(T-t)) - 3$ and $1/\sqrt{T-t}$, respectively. In this case, we cannot rule out that the implied moments depend on maturity, too.
27. See, for example NAKAMURA and SHIRATSUKA (1999).
28. We choose this specification for the dummy to extract a change in skewness or kurtosis which shows some persistence (see the discussion in section 4).

Table 1: Regression results for skewness and excess kurtosis

25. 10.–12. 11. 99: dependent variable: skewness		25. 10.–12. 11. 99 dependent variable: excess kurtosis	
	estimate		estimate
constant	0.570	constant	1.662*
time trend	–0.032	time trend	–0.138*
ECB	–0.239	ECB	–0.477*
17. 01.–11. 02. 99 dependent variable: skewness		17. 01.–11. 02. 99 dependent variable: excess kurtosis	
	estimate		estimate
constant	0.980	constant	–0.528
time trend	–0.054	time trend	0.092*
EUR24	–0.195	EUR24	0.308
EUR27	–0.728*	EUR27	0.206
ECB	0.083	ECB	0.533

Notes: ECB is a dummy variable set equal to one at 4 Nov. and thereafter and at 3 Feb. and thereafter, respectively. EUR24 is a dummy variable set equal to one at 24 Jan. and thereafter and EUR27 is a dummy variable set equal to one at 27 Jan. and thereafter.

* denotes significance at the 5 %-level.

The lower panel of figure 2 shows skewness and excess kurtosis for the subsample from 17 January to 11 February. Skewness averages 0.049 which is substantially below the average of the first subsample indicating more symmetric expectations. Although skewness is positive on most days, on 19 and 27 January and on 9 February it becomes negative with its most negative value below –1.0. Especially on 27 January with the Euro reference value below dollar-parity the median of the distribution shifts in favour of an interest rate increase to the right of the futures implied interest rate. Except for one day, kurtosis is always positive with an average value of 0.648. We regressed skewness and kurtosis on a time trend and individual dummy variables for three events: The drop of the Euro below dollar-parity on 24 January, the Euro reference value below dollar-parity on 27 January, and the official interest rate increase on 3 February. In the skewness regression only the dummy based on the reference value of the Euro exchange rate (EUR27) was individually significant at the 5 %-level in the skewness regression. After that we regressed the skewness measure on a constant, a time trend and all of the three dummies, still retaining a significantly negative coefficient estimate (see table 1). With respect to excess kurtosis none of the dummy variables turned out to be significantly different from zero.²⁹

29. However, the regression results for the skewness parameter appear to depend crucially on the large negative value on 27 January and should be interpreted with care.

5.3. Measures of uncertainty

The degree of uncertainty associated with market participants' expectations of the future interest rate can be captured by a number of different statistics computed from the implied risk-neutral PDF. Apart from the kurtosis measure discussed above the dispersion of expectations can be represented by computing the standard deviation of the implied PDF. However, more common are the implied distribution's percentiles and statistics derived from them.³⁰

Figure 3: Percentiles and scaled interpercentile ranges

Figure 3a: October/November 1999 – March expiration

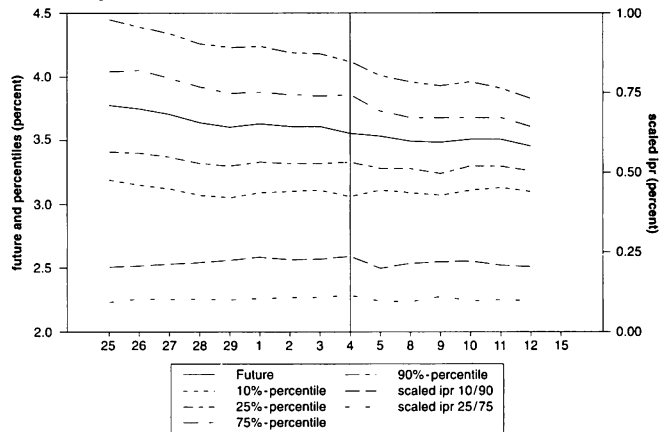
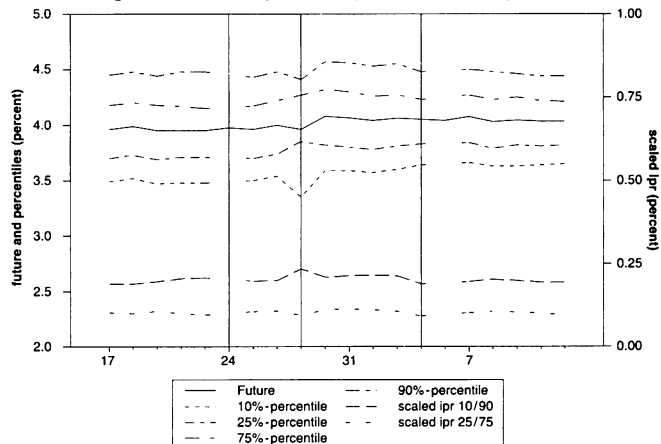


Figure 3b: January/February 2000 – June expiration



30. See, for example BAHRA (1997) and MELICK and THOMAS (1998).

Figure 3 plots the behaviour of the 10, 25, 75 and 90 percent percentiles along with the futures implied interest rate over time. Because the tails of the PDFs approach zero sufficiently fast and levels of interest rates represented by the percentiles are always contained within the range of strike prices we can have some confidence in percentiles derived from the implied PDF.

The top panel exhibits a result typical for percentiles computed with estimated PDFs – maturity dependence. It shows the percentiles converging towards the mean (i.e. the futures implied interest rate) as the option maturity shortens. The intuition behind this result is that, all other things equal, as the time to maturity decreases the market becomes more certain about the future interest rate while extreme outcomes become less likely. The bottom panel does not show the same marked dependence of the percentiles on maturity and might be considered as indicating a period of higher market turbulence compared to the first subsample.

Starting from the calculated percentiles scaled interpercentile ranges (IPR) were constructed by dividing the difference of the 90 and 10 percent percentile, and the 75 and 25 percent percentile, respectively, by the futures implied interest rate. Then, as suggested by MELICK and THOMAS (1998), a constant maturity series of scaled interpercentile ranges was computed for each subsample by regressing the IPR series on a constant and the time to expiration of the option considered. Then, the estimated time trend was subtracted from the IPR resulting in a series with constant maturity equal to that of the option on the last trading day of each subsample. The resulting series are also shown in figure 3.

Table 2: Regression results for scaled interpercentile ranges

25. 10.–12. 11. 99: dependent variable: IPR1090		25. 10.–12. 11. 99 dependent variable: IPR2575	
	estimate		estimate
constant	0.219*	constant	0.102*
ECB	–0.004*	ECB	–0.001
17. 01.–11. 02. 99 dependent variable: IPR1090		17. 01.–11. 02. 99 dependent variable: IPR2575	
	estimate		estimate
constant	0.197*	constant	0.101*
EUR24	0.001	EUR24	–0.006
EUR27	0.018*	EUR27	0.001
ECB	–0.021*	ECB	–0.007**

Notes: IPR1090 is the scaled difference between the 90 and 10 percent percentiles. IPR2575 is the scaled difference between the 75 and 25 percent percentiles. Both series are adjusted for maturity dependence. ECB is a dummy variable set equal to one at the 4 Nov. and thereafter and at Feb. and thereafter, respectively. EUR24 is a dummy variable set equal to one at 24 Jan. and thereafter and EUR27 is a dummy variable set equal to one at 27 Jan. and thereafter.

* denotes significance at the 5 %-, ** at the 10 %-level.

Thinking about the evolution of market sentiment, we would expect the ECB-council's decision to raise interest rates on 4 November either to have a negative effect on the IPR (decrease in uncertainty on the market) or to have no effect at all, because the policy action was widely anticipated. Therefore the IPR series of the first subsample was regressed on a constant and a dummy variable set to one for 4 November and thereafter. The results are shown in table 2. The policy action resulted in a small but statistically significant decline in the scaled difference between the 90 and 10 percent percentile (IPR1090) but no significant effect on that of the 75 and 25 percent percentile (IPR2575). This can be interpreted as being consistent with the results of the preceding sections, where the policy intervention by the ECB had a decreasing effect on uncertainty as measured by IPR, particularly on uncertainty with respect to extreme outcomes for the future interest rate.

For the January/February subsample we would expect to observe an increase in uncertainty with respect to the future development of interest rates to accompany the decline of the Euro exchange rate on 24 and 27 January. The decision of the ECB-council to raise interest rates on 3 February then should have a calming effect on market participants' uncertainty. Again, we regressed the constant maturity IPR series on dummy variables representing the time after the three events: The first decline of the Euro exchange rate below dollar parity, the first decline of the Euro reference value below dollar parity, and the interest rate rise by the ECB. Table 2 shows the results. For the wider percentile (IPR1090) the ECB dummy is found to be significantly negative whereas the dummy variable associated with the reference value of the Euro exchange rate has a significantly positive value. The results for the interquartile range (IPR2575) show only the ECB dummy to be statistically significant (at the 10 % level) with a negative sign but small in absolute value. Again, the ECB's actions reduced market uncertainty and had a comparatively stronger effect on the market's view on extreme outcomes for the Euribor rate.

6. CONCLUSION

In applying a widely used technique to extract a risk-neutral probability distribution function for the level of future interest rates from option prices, a mixture of lognormal densities was fitted to two sets of LIFFE options on three month Euribor futures contracts. The two sets of option prices were chosen to contain important recent events relating to the ECB's monetary policy, namely the decision to raise interest rates in November 1999 and the increase in official rates in February 2000 following the drop of the Euro exchange rate below dollar-parity.

The results were found to be consistent with market comment at that time. Firstly, the ECB-council's decision in November 1999, that indeed appears to be anticipated by market participants, nevertheless was awaited with considerable uncertainty among traders which was, in turn, significantly reduced after the decision was announced. Sec-

only, the decline in the reference value of the Euro below dollar-parity on average made market participants expect an increase in interest rates but also led to increased uncertainty with respect to the future evolution of the Euribor. The ECB-council meeting with its decision to raise official interest rates again had a calming effect on the level of uncertainty on the market.

It was shown that the observed changes in uncertainty predominantly were present in the tails of the estimated PDF, thus being related to extreme outcomes for the Euribor rate. Monetary policy actions did not have much effect on the probability densities associated with more plausible interest rate movements. This indicates that, apart from some extreme scenarios with little probability, the ECB's actions both times were anticipated fairly accurately. Furthermore, expectations of an interest rate increase became more pervasive after the reference value of the Euro exchange rate declined below one US-Dollar on 27 January. This provides evidence that, notwithstanding the statements of prominent members of the ECB-council of not making monetary policy decisions dependent on the exchange rate, market participants expected the ECB-council to react to the Euro's depreciation by raising interest rates.

However, some caveats should be noted. First, the estimated risk-neutral PDF will deviate from the true density of market participants' expectations in the presence of risk aversion. As long as only changes in the shape of the estimated density are investigated, this problem is mitigated by focussing only on comparisons of PDFs derived from option prices within short time windows. Then risk aversion can be assumed to remain unchanged and changes in the PDF can be interpreted as providing insights into changes of the true density (MELICK and THOMAS, 1998). Second, the mixture of lognormals, although not very restrictive, is only one possible assumption to provide some structure on the implied PDF. As MELICK and THOMAS (1995, 1998) note, there are many densities observationally equivalent to the lognormal mixture, i.e. those are able to generate the same set of observed option prices. Finally, in interpreting the changes in the shapes of the estimated risk-neutral PDFs many subjective judgements are involved (CHANG and MELICK, 1999). Amending the informal inspection of the estimated PDFs over time with some simple regressions, this study tried to obtain a more objective perspective on the changes in implied PDF with respect to some important events. Nevertheless, many questions remain. Particularly important are further investigations into the question when a change in the shape of the PDF and in the derived statistics can be considered to be significant compared to the "normal" changes.

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SUMMARY

In recent years various different techniques to uncover the information on market expectations contained in option prices have been developed. This paper applies the technique of fitting a mixture of lognormal densities to LIFFE Euribor futures options to estimate the risk-neutral implied probability density function for the future level of interest rates. Two sets of option prices are considered which cover the ECB's increases in official interest rates in November and February. The results are found to be consistent with market comment prevailing at that time.

ZUSAMMENFASSUNG

In den vergangenen Jahren wurde eine Reihe neuerer Verfahren zur Ermittlung der Markterwartungen auf Finanzmärkten aus beobachteten Optionspreisen entwickelt. In dieser Arbeit wird ein auf diesem Gebiet verbreiteter Ansatz auf LIFFE Euribor Futures-Optionen angewendet, indem die Parameter einer Mischung von Lognormalverteilungen aus beobachteten Optionspreisen geschätzt werden. Die so gewonnenen risikoneutralen Dichtefunktionen werden benutzt, um einen Einblick in die Entwicklung der Markterwartungen um die Zeiträume der Zinserhöhungen durch die EZB im No-

vember 1999 und Februar 2000 zu gewinnen. Die Ergebnisse zeigen deutliche Übereinstimmungen mit den Kommentaren von Marktbeobachtern zu diesen Zeitpunkten.

RÉSUMÉ

Au cours des années passées, de nouvelles méthodes servant à détecter les attentes des acteurs sur les marchés financiers ont été développées. Ces méthodes se basent sur l'observation des prix des options. Dans cette contribution, une méthode courante dans ce domaine est appliquée aux options-futures LIFFE Euribor: les paramètres d'un mélange de distributions log-normales sont estimés à partir de prix d'options observés. Les fonctions de densité neutres au risque ainsi obtenues sont utilisées afin d'obtenir des informations sur l'évolution des attentes du marché au moment des hausses de taux d'intérêt par la BCE en novembre 1999 et en février 2000. Les résultats correspondent clairement aux commentaires des observateurs du marché à cette époque.