

Nonlinear adjustment towards purchasing power parity: the Swiss Franc-German Mark case

ROGER GUERRA*

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1. INTRODUCTION

Purchasing power parity (PPP) has been a central issue in economics for many years. PPP states that the price of a basket of (tradable) goods should be the same across all countries, when transportation costs and tariffs are disregarded. Therefore we can write:

$$P = EP^* \quad (1)$$

where P , E and P^* represent the domestic price level, the domestic price of one foreign monetary unit and the foreign price level, respectively. Taking logs and adding the time index gets

$$e_t = p_t - p_t^*. \quad (2)$$

PPP is at the centre of a broad literature which so far has not provided precise answers as to whether it holds or not.¹ This lack of empirical support has led some researchers to testing of the hypothesis that the real exchange rate q_t ($q_t = e_t - p_t + p_t^*$) follows a random walk. Examples can be found in ADLER and LEHMANN (1983), HUIZINGA (1987) and MEESE and ROGOFF (1988). The development of cointegration techniques has brought a new way of testing PPP. The main purpose of this kind of studies is to check for the existence of a long-run equilibrium relationship between domestic and foreign prices and the nominal exchange rate, assuming that in the short run PPP does not hold. Among the plethora of papers applying cointegration, we may point out CORBAE and OULIARIS (1988), MARK (1990) and CHEUNG and LAI (1993). Results based on this technique are somewhat less negative than those from earlier studies, but they are still not convincing. Moreover, as showed by KUGLER and LENZ (1993), using a multivariate cointegration framework tends to produce results more in line with PPP. Thus, the empiri-

* University of Geneva, Department of Economics, Bd. du Pont d'Arve 40, 1211 Geneva 4. E-mail: Guerra@ecopo.unige.ch. I would like to thank Ulrich Kohli, Peter Kugler, Giovanni Ferro-Luzzi and Claudio Sfreddo for their helpful comments. Of course any remaining errors are my own.

1. See for instance FROOT and ROGOFF's (1995) review.

cal technique could introduce an additional bias in the tests. More recent papers (LOTHIAN and M.P. TAYLOR, 1996; TAYLOR A.M., 2001; LOTHIAN and M.P. TAYLOR, 2000) use long-horizon data to test purchasing power parity. Their results, based on two-century panel data, suggest that PPP may hold over the long-run. Again, there is no general agreement on these results, because using the same data, CUDDINGTON and LIANG (2000) do not reject the hypothesis of unit root of the dollar/sterling real exchange rate when high-order lags or a time trend are included in the augmented Dickey-Fuller (ADF) test. A similar conclusion is reached by O'CONNELL (1998) and ENGEL (2000), who emphasize the weak power of unit root tests.

This contradictory empirical evidence has given an impulsion to the theoretical analysis of the "PPP problem". The attention has focused on the inclusion of transaction costs in exchange rate models. Examples of this kind of work are BENNINGA and PROTOPAPADAKIS (1988), DUMAS (1992) and SERCU, UPPAL and VAN HULLE (1995). DUMAS' model embodies proportional transaction costs into a spatially separated markets model. The equilibrium of the model implies that deviations from the PPP are not mean-reverting as long as they are small (compared to the cost of exchanging the goods).² If the deviations become large enough (as a consequence of an exogenous shock), parity is reached very quickly. The speed of adjustment towards purchasing power parity should depend on the size of the deviation from it. The dynamic process of the real exchange rate then is nonlinear. Small deviations give divergent behaviour, whereas large deviations involve a rapid mean-reverting behaviour. DUMAS' model makes it possible to explain the findings of previous work, that is, that the real exchange rate is a random walk (for small deviations).

In a remarkable paper on the six major international macroeconomic puzzles, OBSTFELD and ROGOFF (2000) argue that the model of DUMAS can explain the "PPP puzzle", but it fails to provide a clear explanation of the real exchange rate behaviour within the no-arbitrage band. We leave aside this problem and focus on the empirical issue, in order to detect the nonlinearity of the deviations. For further details on the theoretical question, we refer the reader to OBSTFELD and ROGOFF's article.

To test the conclusions of the Dumas' model we will use, as suggested by MICHEAL, NOBAY and PEEL (1997), a smooth transition autoregressive (STAR) representation of the deviation from the parity. This kind of model was developed by GRANGER and TERÄSVIRTA (1993) and TERÄSVIRTA (1994). The STAR representation may be more suitable than a threshold autoregressive model (TAR) because, as suggested by Dumas, transaction costs create a band of no-arbitrage which can vary depending on the mix of goods exported and imported by a pair of trading partners. The fixed thresholds of a standard TAR model cannot take account of the possible variations of the band, and is thus more appropriate in a context with explicit bands like the EMS exchange rate mechanism.

The next section provides an explanation of the STAR model, while section 3 reports data description and estimation's results. Section 4 gives some concluding remarks.

2. Within the so-called "no-arbitrage" zone for the real exchange rate.

2. MODELLING NONLINEAR ADJUSTMENT

To represent the nonlinear adjustment of the purchasing power parity implied by Dumas' model we will implement a STAR model.³ This group of models is a generalisation of the exponential autoregressive model (EAR) of HAGGAN and OZAKI (1981) and the threshold autoregressive model (TAR). It allows for the transition between two regimes, which depends on the extent of the deviation, and where adjustments take place every period.

The general formula of the model is

$$y_t = \theta_{10} + \theta'_1 w_t + (\theta_{20} + \theta'_2 w_t)G(y_{t-d}; \gamma, c) + u_t \quad (3)$$

where $\{y_t\}$ is a stationary and ergodic process, a p -dimension vector of lagged values of $\{y_t\}$ and $u_t \sim \text{iid}(0, \sigma^2)$ is the transition function which depends on y_{t-d} , the transition variable, the adjustment speed $\gamma > 0$, and the equilibrium parameter c . GRANGER and TERÄSVIRTA (GT) (1993) suggest two possible transition functions, namely

$$G_1(y_{t-d}; \gamma, c) = \{1 + \exp[-\gamma(y_{t-d} - c)]\}^{-1} \quad (4)$$

$$G_2(y_{t-d}; \gamma, c) = 1 - \exp[-\gamma(y_{t-d} - c)^2] \quad (5)$$

where the first is a logistic function (LSTAR) and the second is an exponential function (ESTAR). G_1 is asymmetric, the two extreme (outer) regimes are $G_1 = 0$ when $y_{t-d} - c \rightarrow -\infty$, and $G_1 = 1$ when $y_{t-d} - c \rightarrow +\infty$. The inner regime occurs when $y_{t-d} \approx c$, so that $G_1 = \frac{1}{2}$. The exponential transition function G_2 is symmetric around c , the outer regime is given by $G_2 = 1$ for $y_{t-d} - c \rightarrow \pm\infty$, and the inner regime is given by $G_2 = 0$ when $y_{t-d} \approx c$. The choice of the transition function is made using a battery of nested tests as proposed by GT. However, in our example the appropriate model is the ESTAR which involves a symmetric behaviour of the adjustment whether the deviation is positive or negative.

The two extreme regimes implied by the STAR model ($G_i = 0$ and $G_i = 1$ ($i = 1, 2$)) generate two different $\text{AR}(p)$ models

$$G_i = 0 : y_t = \theta_{10} + \theta'_1 w_t + u_t \quad (6)$$

$$G_i = 1 : y_t = (\theta_{10} + \theta_{20}) + (\theta'_1 + \theta'_2)w_t + u_t. \quad (7)$$

While $\theta_{1i} \geq 1$ ($i = 1, \dots, p$) is possible (θ_{1i} is the i 'th element of the θ_1 coefficients' vector), we need $(\theta_{1i} + \theta_{2i}) < 1$ for the model to be stable. For small values of $y_{t-d} - c$ the

3. The classical techniques (LSQ, ARMA, cointegration) do not allow for a non-linear adjustment.

corresponding $AR(p)$ model (6) is either explosive or presents a unit root when $\theta_{1i} \geq 1$, justifying the hypothesis of random walk suggested by previous empirical work. If the deviations from the equilibrium are large and if the above condition on the coefficients holds, the disequilibrium comes down to (7) at a speed dictated by the parameter γ .

To assess the adequacy of a STAR model, we need to test the linearity of y_t . First we test for cointegration. If a cointegration relation is found between the variables of the model considered (in our case e, p and p^*) we can infer that y_t adjusts linearly and at a constant speed. On the other hand, if no cointegration is detected, a test of linearity is given by $H_0 : \gamma = 0$, which implies that y_t is described by a linear $AR(p)$ model because $G_i = 0$. It can be seen that (3) is only identified under the alternative $H_1 : \gamma > 0$, since the parameters c and θ_{2i} ($i = 0, 1, \dots, p$) can take any value if $\gamma = 0$. To overcome this problem we will apply the approach suggested by LUUKKONEN, SAIKKONEN and TERÄSVIRTA (1988), who derive an auxiliary regression of (3) by taking a third-order TAYLOR approximation of G_1 :⁴

$$y_t = \theta_{10} + \theta'_1 w_t + (\theta_{20} + \theta'_2 w_t)T_3 + u_t \quad (8)$$

where

$$T_3 = \delta_0 + \delta_1 y_{t-d} + \delta_2 y_{t-d}^2 + \delta_3 y_{t-d}^3 + R_3. \quad (9)$$

R_3 represents the remainder of the TAYLOR approximation. The auxiliary regression becomes

$$y_t = \alpha + \beta'_0 w_t + (w_t y_{t-d})' \beta_1 + (w_t y_{t-d}^2)' \beta_2 + (w_t y_{t-d}^3)' \beta_3 + u_t^*, \quad (10)$$

where u_t^* is a function of u_t and β_j ($j = 1, 2, 3$) are functions of the parameters of (3). The null hypothesis of nonlinearity (supposing d known) is given by

$$H_0 : \beta_1 = \beta_2 = \beta_3 = 0, \quad (11)$$

against the general alternative H_1 : “at least one $\beta_j \neq 0$ ”. The corresponding test statistic is a Lagrange multiplier-type (LM) that follows a χ^2 distribution with $3p$ degrees of freedom. To improve the power of the linearity test we will apply a Fisher-test as suggested by GT. We will call this test F_L . The F_L is a test of linearity against the alternative that y_t is described by a STAR model.

If linearity is rejected, we have to select the appropriate STAR model. To do this, we will implement the GRANGER and TERÄSVIRTA algorithm. The procedure is based on the auxiliary regression (10). The third-order TAYLOR approximation of the logistic function includes third-order powers of the variables, whereas the approximation of the exponen-

4. The third-order TAYLOR approximation of the logistic transition function encompasses the third-order TAYLOR approximation of the exponential function.

tial contains only second-order powers terms.⁵ So, a first test (called F_3) of the ESTAR against the LSTAR specification is $H_0 : \beta_3 = 0$. If the null is not rejected, a more powerful test (called F_{ES}) for the ESTAR specification against LSTAR and linearity is $H_0 : \beta_2 = 0 | \beta_3 = 0$. Rejection of the latter hypothesis implies acceptance of the ESTAR specification.

By testing linearity and the specification of the model we have assumed that d , the delay parameter, is known. To determine the delay of the transition variable we will follow the procedure proposed by TERÄSVIRTA (1994). He suggests to implement the linearity test for many values of d , and then to choose the delay parameter which minimises the p -value of the corresponding F_L test.⁶

The autoregressive order p is chosen based on the partial autocorrelation function (PAC) instead of using the information criterion such as the Akaike (AIC) or the Schwarz (SBC) criterion. This is due to the well-known underspecification characteristics of the AIC or the SBC criterion. To test linearity it is more suitable to overspecify p , because underspecification can bias the results of the tests, as argued by GT.

3. EMPIRICAL ANALYSIS

We will analyse the Swiss case, by testing for purchasing power parity with respect to Germany. PAPELL (1997) weakly rejects the unit root hypothesis of the real exchange rate and PIPPENGER (1993) found evidence of cointegration using 1973–88 data. We try to find more convincing results using a nonlinear approach. We use quarterly data in logarithms spanning from the first quarter 1960 to the second quarter 1998. This sample includes both a Bretton Woods fixed exchange rate period and a floating rate experience but we also carry out estimations based only on the floating period. The price level for both countries are represented by the consumer price index (CPI). The exchange rate and the Swiss price index data were obtained from the Swiss National Bank, and the German price index from the OECD Statistic Compendium.

3.1. The 1960–1998 period

The three series have been tested for stationarity; the results (not reported) indicate that all can be considered $I(1)$. Cointegration tests have been carried out using JOHANSEN (1988) trace- and $\lambda_{\max}$ -test. The selected model specification for the cointegration relation is⁷

$$e_t = \mu + \alpha p_t + \beta p_t^* + y_t \quad (12)$$

5. See LUUKKONEN, SAIKKONEN and TERÄSVIRTA (1988) for the complete derivation of this result.
6. If $pv(d)$ is the p -value which corresponds to the $F_L(d)$ test, where $1 \leq d \leq D$, then $\hat{d}$ (the chosen delay value) is obtained as $pv(\hat{d}) = \min_{1 \leq d \leq D} pv(d)$.
7. We applied the Pantula's principle (see for instance HARRIS, 1995).

where y_t represents the deviations from parity (and the real exchange rate) and where we test the crucial hypothesis $\alpha = -\beta = 1$. The statistics of the JOHANSEN's tests for the null of no cointegration vector are $\lambda_{trace} = 32.5$ and $\lambda_{max} = 14.34$. Thus, the null cannot be rejected at the 5 % significance level. This result implies the rejection of the hypothesis of a long run linear adjustment. The residuals from the estimation are used to test linearity against STAR specification.

In the previous section, we indicated that the process described by the STAR model should be stationary. Application of an ADF test on the residuals y_t yields a test statistic $\tau = -4.01$ which is significant at the 5 % level. Variable y_t is thus stationary. Figure 1 reveals that the highest significant lag is the fifth, so we set the autoregressive order p at 5. Table 1 presents the results from the estimation of the linear model (AR(5)). Results of the diagnostic tests show that the residuals exhibit no autocorrelation, which means that the number of lag in the regression is not underspecified. An ARCH effect up to order 4 is detected by the ENGLE (1982) test and the null of normality is rejected.

Figure 1: PAC function (1960–1998)

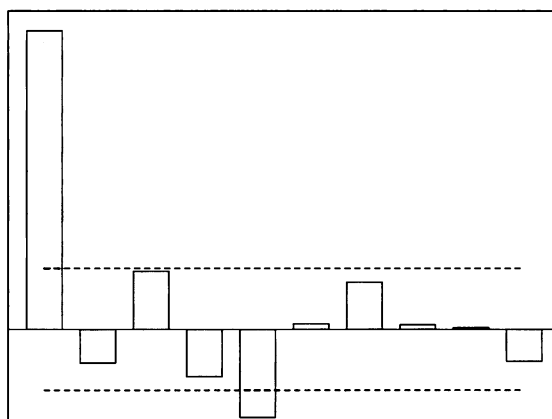


Table 1: Linear autoregressive model

y_t	$= -0.015$	$+ 0.92 y_{t-1}$	$- 0.22 y_{t-2}$	$+ 0.214 y_{t-3}$	$+ 0.118 y_{t-4}$	$- 0.263 y_{t-5}$
	(0.004)	(0.08)	(0.109)	(0.109)	(0.109)	(0.081)
$R^2 = 0.706, \quad \sigma = 0.031, \quad Q(4) = 0.356, \quad Q(12) = 12.1,$						
$J.B. = 18.05^{**}, \quad A(1) = 5.68^*, \quad A(4) = 9.48^*.$						

Notes: Numbers in parentheses are standard deviations. σ represents the standard deviation of the regression; $Q(k)$ is the Ljung-Box statistic for residual autocorrelation up to order k ; $J.B.$ is the Jarque-Bera normality test statistic; $A(k)$ is the ENGLE (1982) test for ARCH effect up to order k in the residuals. The $J.B.$ test statistic is distributed as a χ^2 with 2 d.f., while the $Q(k)$ and the $A(k)$ statistics are distributed as χ^2 with k d.f. * Significant at the 0.05 level; ** Significant at the 0.01 level.

In the light of the poor performance of the linear model, we tested for linearity. We estimated the following auxiliary regression

$$y_t = \alpha + \sum_{i=1}^5 \beta_{0i} y_{t-i} + \sum_{i=1}^5 \beta_{1i} y_{t-i} y_{t-d} + \sum_{i=1}^5 \beta_{2i} y_{t-i} y_{t-d}^2 + \sum_{i=1}^5 \beta_{3i} y_{t-i} y_{t-d}^3 + u_t^*. \quad (13)$$

The null of linearity is $H_0 : \beta_{1i} = \beta_{2i} = \beta_{3i} = 0, 1, \dots, 5$. Test for values of d 1 to 8 ($D = [1, \dots, 8]$) was carried out. The choice of the delay parameter was made based upon the p -values of the F_L test statistics, which can be found in Table 2.

Table 2: Linearity and specification tests

d	1	2	3	4	5	6	7	8
F_L	0.0002	0.377	0.306	0.082	0.015	0.031	0.005	0.787
F_3	0.108	0.661	0.658	0.075	0.447	0.029	0.023	0.959
F_{ES}	0.006	0.483	0.585	0.260	0.241	0.388	0.019	0.296

The null of linearity is strongly rejected for $d = 1, 5, 6, 7$. Using the procedure suggested by Teräsvirta, we chose $\hat{d} = 1$, the transition variable is thus y_{t-1} . The clearcut rejection of the linearity enables us to test for the STAR model specification. The P -value of the F_3 test shows that the third-order power of the auxiliary regression can be considered non significant ($\beta_{3i} = 0$), whereas the null of the nested F_{ES} ($\beta_{2i} = 0 | \beta_{3i} = 0$) test is unambiguously rejected. We infer from the testing procedure that the deviations from the purchasing power parity for the Switzerland can be described through an ESTAR model.

The estimation outcome of the ESTAR model using nonlinear least squares is reported in Table 3. The same results are obtained by varying the starting values, thus suggesting a global minimum.⁸ The parameters are of the expected sign and size, particularly the crucial coefficients ($\gamma, c, \theta_{11}, \theta_{21}$). The magnitude speed of adjustment, γ , indicates a very rapid adjustment when the deviation becomes large enough.⁹

Table 3: The ESTAR model^a

$y_t = -0.024 + 1.257 y_{t-1} - 0.234 y_{t-2} - 0.113 y_{t-3} + 0.446 y_{t-4} - 0.648 y_{t-5} +$ (0.022) (0.212) (0.172) (0.20) (0.191) (0.142)	
$\left(\begin{array}{cc} 0.023 & -0.699 y_{t-1} - 0.015 y_{t-2} + 0.712 y_{t-3} - 0.950 y_{t-4} + 1.04 y_{t-5} \end{array} \right) \hat{G}_E$ (0.027) (0.276) (0.307) (0.367) (0.418) (0.317)	
$\hat{G}_E = 1 - \exp \left[\frac{-144.61}{(85.77)} \left(y_{t-1} + \frac{0.132}{(0.001)} \right)^2 \right]$	
$R^2 = 0.769, \quad \sigma = 0.28, \quad Q(4) = 0.622, \quad Q(12) = 12.6,$	
$J.B. = 1.028, \quad A(1) = 0.009, \quad A(4) = 0.706,$	
$LR_A = 1.078, \quad LR_B = 10.246^{**}$	

Notes: ^a See the notes for Table 1; ** Significant at 0.01 level.

8. GRANGER and TERÄSVIRTA (1993) suggest standardizing the transition function exponent by the sample variance of y_t in order to choose a consistent starting value. In our estimation procedure, we applied a wide range of starting values for γ , which always converged to the same estimates.
9. The same order of magnitude has been found by MICHAEL, NOBAY and PEEL (1997).

The diagnostic tests¹⁰ show that the non-normality and the heteroskedasticity of the residuals have disappeared, meaning that the nonlinear specification is more appropriate. The gain in terms of residual variance is 16 %. We tested two hypothesis (*A* and *B*) on the crucial parameters θ_{11} and θ_{21} using likelihood ratio-type tests. The null are $H_{0A} : \theta_{11} = 1$ and $H_{0B} : \theta_{11} + \theta_{21} = 1$. Results reported in Table 3 show that H_{0A} cannot be rejected, while H_{0B} is rejected at the 1 % significance level. The rejection of the latter hypothesis enables us to conclude that in the case of large deviations the model is stable. We infer that for small deviations ($y_{t-1} \approx -.132$), the real exchange rate is a random-walk process. However, as the deviation becomes larger the real exchange exhibit clear mean-reverting behaviour.

In the PPP literature a commonly used measure of the deviation persistence is the deviation half-life (H). The half-life is usually computed from a simple $AR(1)$ model.¹¹ Our steady state model is an $AR(5)$ model, hence the calculation of H is not an easy task. To overcome this problem, and to get an approximation of H , we have simulated the effects of an unit shock at period $t - j$ on the deviation y_t (this effect is called the dynamic multiplier). Considering a 6-quarter lag ($j = 6$), our multiplier is .502 and for $j = 7$ it is .089. We can infer that the deviation half-life is about 1.5 years. This value is lower than the common result of a half-life of about 4–5 years. An explanation of this peculiar behaviour is given by TAYLOR A.M. (2001), who argues that taking account of nonlinearity in the deviation makes it possible to reduce the half-life bias.

An interesting remark can be made on the basis of the transition function $\hat{G}_E$, which is represented in the Figure 2 against the deviation from the equilibrium $y_{t-1} - c$. The nonlinearity of the transition is unambiguous, although it should only be taken as indicative. We have a reasonable number of observations above and below the equilibrium ($c = -.132$). Thus, the ESTAR specification seems to be a good choice. The transition function takes values between 0 and 1, therefore the two extreme regimes are represented in our sample.

The coefficient c could be viewed as the equilibrium value of the real exchange rate. A corresponding nominal exchange rate can be calculated by fixing a reference period. The value of the “equilibrium nominal exchange rate” for the second quarter of 1998 is .781 francs for a German mark. This figure is below the limit of .80 francs per mark set by the Swiss National Bank (SNB) in the late 70’s as the maximum acceptable value of the Swiss franc in order to avoid a slowdown of Swiss exports. The observed nominal exchange rate at period 98:2 is .842 which implies a clear undervaluation of the Swiss franc towards the PPP.

10. See the notes to Table 1.

11. If the the $AR(1)$ model is $y_t = \rho y_{t-1} + \varepsilon_t$
then half-life is calculated as $H = \frac{\ln \frac{1}{2}}{\ln \rho}$

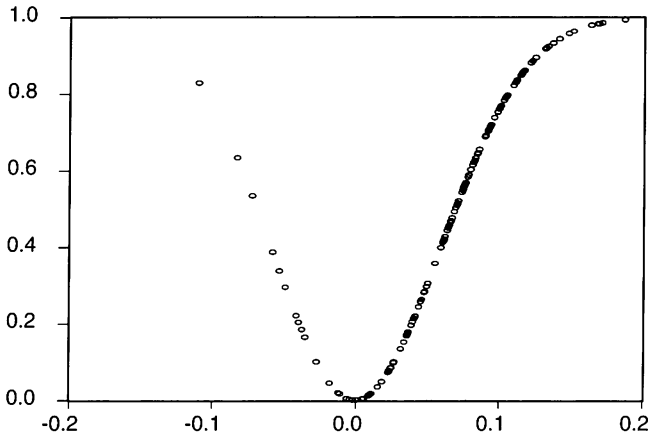
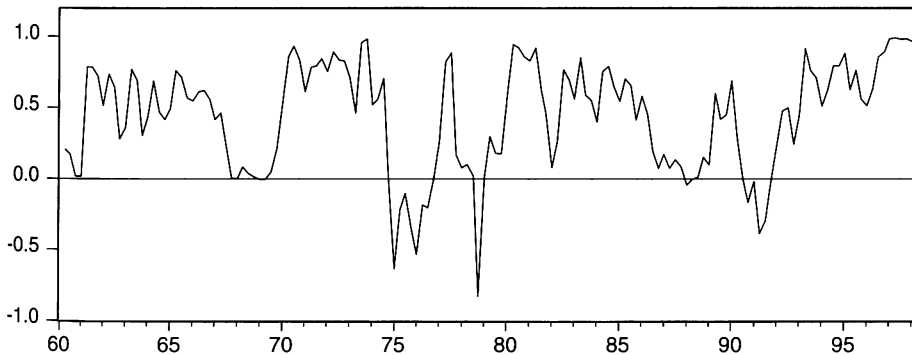
Figure 2: Transition function

Figure 3 shows the transition function adjusted for the sign of the estimated equilibrium deviations $(y_{t-1} - c)$, hence negative (positive) values indicate overvaluation (undervaluation) of the Swiss franc relative to PPP.¹² The negative values of $y_{t-1} - c$ correspond to period of economic turmoil such as the 70's oil crises and the 1991 Gulf War. We deduce that during these periods the Swiss franc was overvalued vis-à-vis its purchasing power parity. It is often thought that in trouble periods the Swiss franc is considered as a safe haven instrument.

Figure 3: Over- and under-valuation of the Swiss franc

12. The exact formulation is $\Gamma(y_{t-1} - c) = \left\{ 1 - \exp \left[-\gamma (y_{t-1} - c)^2 \right] \right\} \operatorname{sgn} (y_{t-1} - c)$, where $\operatorname{sgn} (y_{t-1} - c) \equiv (y_{t-1} - c) / |(y_{t-1} - c)|$

We have reestimated the model without the non significant variables. This is a good exercise to check the robustness of our results. Table 4 reports the restricted estimation outcome. It is clear that the omission of the non-significant variables has little impact on the estimates of the coefficients.

Table 4: Restricted ESTAR model^a

$y_t = -1.439 y_{t-1} - 0.282 y_{t-2} + 0.354 y_{t-4} - 0.602 y_{t-5} +$
$(0.116) \quad (0.093) \quad (0.132) \quad (0.125)$
$\left(-0.801 y_{t-1} + 0.580 y_{t-3} - 0.807 y_{t-4} - 0.999 y_{t-5} \right) \hat{G}_E$
$(0.161) \quad (0.201) \quad (0.351) \quad (0.263)$
$\hat{G}_E = 1 - \exp \left[-\frac{176.7}{(83.15)} \left(y_{t-1} + \frac{0.124}{(0.008)} \right)^2 \right]$
$R^2 = 0.764, \quad \sigma = 0.28, \quad Q(4) = 0.578, \quad Q(12) = 12.56.$
$J.B. = 0.633, \quad A(1) = 0.045, \quad A(4) = 0.698.$

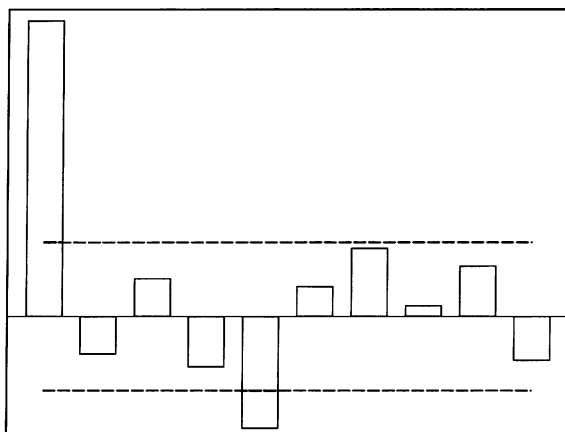
Note: ^a See the notes for Table 1.

3.2. The post Bretton-Woods period

The empirical analysis in the previous subsection was carried out on a sample including fixed and floating exchange rates regimes. The results obtained may be sensitive to the exchange rate regime switch. However, as pointed out by TAYLOR A.M. (2002), the floating exchange rate period exhibits larger deviations from PPP, even if these results are not caused by significantly greater persistence of deviations but is due to larger shocks to the real exchange rate during this period. In order to assess the impact of the exchange rate regime on the adjustment of real exchange rate deviations and to compare our results with similar studies,¹³ we implement the nonlinear analysis on a reduced sample spanning from the first quarter 1973 to the second quarter 1998.

The cointegration tests ($\lambda_{trace} = 26.86$ and $\lambda_{max} = 17.43$) lead us not to reject the null of no cointegration and hence refute the long-run linear adjustment. The ADF test on the residuals y_t of the linear relationship (12) yields a significant (1% level) statistic $\tau = -3.62$. Figure 4 shows the partial autocorrelation function of y_t . It is straightforward that the autoregressive order is $p = 5$. The linear estimation results are presented in Table 5.

13. BAUM ET AL. (2001) and KILIAN and M.P. TAYLOR (2001) investigate, among other currencies, the Swiss PPP relative to the US dollar.

Figure 4: PAC function (1973–1998)**Table 5: Linear autoregressive model (1973–1998)^a**

$$y_t = -0.018 + 0.881 y_{t-1} - 0.183 y_{t-2} + 0.22 y_{t-3} + 0.093 y_{t-4} - 0.276 y_{t-5}$$

$$(0.006) \quad (0.099) \quad (0.135) \quad (0.134) \quad (0.134) \quad (0.102)$$

$$R^2 = 0.69, \quad \sigma = 0.035, \quad Q(4) = 0.437, \quad Q(12) = 10.2,$$

$$J.B.^* = 4.57, \quad A(1) = 1.99, \quad A(4) = 7.75^*.$$

Notes: ^a See the notes for Table 1. * Significant at the 0.1 level.

The results from the linear $AR(5)$ are better than those found with the entire sample: no autocorrelation in the residuals up to twelve lags is detected while the normality assumption and no ARCH effects of order four are rejected at a low 10 % significance level. However, so as to complete the analysis we carry out the linearity test. Table 6 presents the p -values of the linearity test F_L and the model selection tests F_3 and F_{ES} . Non-linearity is clearly detected for $d = 1, 5$ and 7. We choose $\hat{d} = 1$, which minimizes the p -values of the linearity test. For this delay parameter, the F_3 test is not significant while the null hypothesis of the F_{ES} test is clearly rejected. We thus infer that the deviations from the PPP during the post Bretton-Woods period can be represented by an exponential STAR model with y_{t-1} as transition variable.

Table 6: Linearity and specification tests (1973–1998)

d	1	2	3	4	5	6	7	8
F_L	0.003	0.647	0.256	0.222	0.034	0.095	0.015	0.892
F_3	0.174	0.400	0.107	0.052	0.004	0.201	0.023	0.968
F_{ES}	0.024	0.755	0.662	0.545	0.375	0.223	0.069	0.347

The estimation of the nonlinear ESTAR model is obtained by nonlinear least squares. The outcome is reported in Table 7. Once again the sign and size of the parameters are as expected. The speed of adjustment, larger than that found previously, is $\gamma = 167.58$ which means that for sizeable deviations from the equilibrium the adjustment takes place rapidly.¹⁴ The gain in terms of residual variance relative to the linear model is about 20 %, higher than the 16 % found previously.

Table 7: The ESTAR model (1973–1998)^a

$y_t = -0.027 + 1.252 y_{t-1} - 0.167 y_{t-2} - 0.250 y_{t-3} + 0.553 y_{t-4} - 0.717 y_{t-5} +$						
(0.030)	(0.276)	(0.208)	(0.254)	(0.242)	(0.177)	
$\left(\begin{array}{cccccc} 0.027 & -0.711 & -0.127 & 0.963 & -1.24 & 1.21 \\ (0.036) & (0.324) & (0.359) & (0.449) & (0.511) & (0.395) \end{array} \right) \hat{G}_E$						
$\hat{G}_E = 1 - \exp \left[- \frac{167.58}{(102.3)} \left(y_{t-1} + \frac{0.135}{(0.01)} \right)^2 \right]$						
$R^2 = 0.773, \quad \sigma = 0.031, \quad Q(4) = 0.572, \quad Q(12) = 6.67,$						
$J.B. = 0.104, \quad A(1) = 0.200, \quad A(4) = 2, 41,$						
$LR_A = 0.973, \quad LR_B = 4.06^*$						

Notes: ^a See the notes for Table 1. * Significant at 0.05 level.

The diagnostic tests show that the estimated ESTAR model seems to be consistent. The hypothesis $H_{0A} : \theta_{11} = 1$ cannot be rejected, which confirms that, for small deviations from the PPP, the real exchange rate follows a random walk. On the other hand, the second hypothesis tested, $H_{0B} : \theta_{11} + \theta_{21} = 1$, is rejected. Thus, for larger deviations the real exchange rate is mean-reverting. In the previous subsection, we have evaluated the persistence of the deviation from the PPP by approximating the half-life of the deviation. We found a half-life of about 6 quarters. Repeating the same experiment for the reduced sample, we find that the ninth quarter dynamic multiplier is 0.515 while the tenth quarter multiplier is clearly smaller. We thus deduce that the approximated half-life of the deviation is just smaller than 2.5 years. This value is just smaller than the findings of KILIAN and M.P. TAYLOR (2001) where the half-life varies between 5 and 16 quarters depending on the shock size for the Swiss franc / US dollar real exchange rate. The increase in the half-life found in our floating period analysis can be considered as a confirmation of higher volatility of nominal exchange rates during this interval.

It is worth noting that the parameters found are fairly similar to those obtained using the longer sample. This result confirms the finding of TAYLOR A.M. (2002) which points out that, over the twentieth century, and thus over several exchange rate regimes including fixed and floating ones, there was little changes in the capacity of international market integration to smooth out real exchange rate shocks.

14. KILIAN and M. P. TAYLOR (2001) obtain a standardized value of γ equal to 0.724 for the franc-dollar parity. Multiplying our estimated value by the sample variance of y_t we get 0.6155.

3.3. Impulse-response function

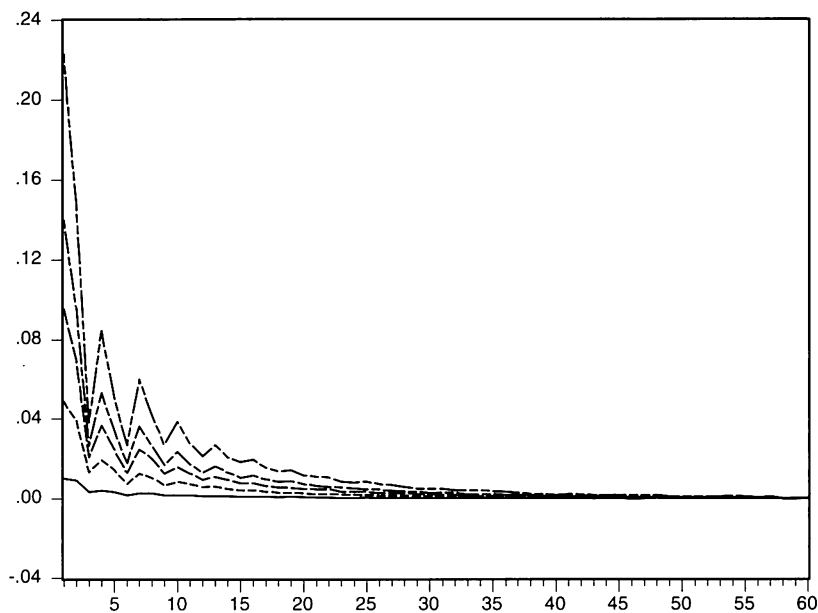
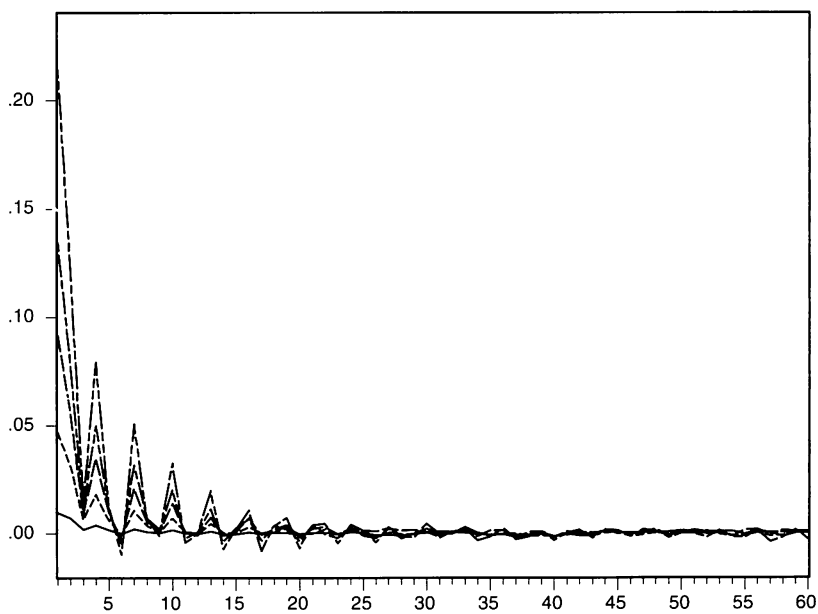
In the previous subsection we discussed the model stability and the mean reverting behaviour resulting from important deviations. To assess rigorously the degree of mean reversion and stability, we implement a dynamic stochastic simulation. The standard tool used in such a situation is the impulse response function. The impulse response function (IRF) allows to describe the dynamic adjustment following a temporary initial shock. The IRF is usually used in a linear context where it does not depend on initial conditions, future innovations and the size of the shocks. In a nonlinear framework all these elements influence the value of the function (GALLANT et al., 1993; KOOP et al., 1996).

We define the impulse response function as a sequence of N values $\{I(1, c, h), I(2, c, h), \dots, I(N, c, h)\}$ representing the difference between the expected evolution of y on the period $t + 1$ and $t + N$ with and without an additive shock of size c , occurring at period t , conditioned on the history of the system h at period t :

$$I[j, c(t), h(t)] = E[y_{t+j}|c(t), h(t)] - E[y_{t+j}|h(t)], \quad j = 1, 2, \dots, N.$$

$I[\cdot]$ is calculated by Monte Carlo simulation using our data set. The first p observations $(1, 2, \dots, p)$ of the sample are used as initial condition of our model $(y_{t-p+1}, y_{t-p+2}, y_t)$. 200 sets of residuals are simulated over N periods (from $t + 1$ to $t + N$). We calculate and store the difference of the simulated values of the deviations y with and without a shock c using our estimated model. We fix a new set of initial conditions by moving up by one observation in our sample $(2, 3, \dots, p + 1)$ and we proceed with a new simulation. This procedure is repeated for all the observations of the sample. Taking the mean over all the sets of difference of the expected deviations, we obtain our impulse response function conditioned on the available information.

We applied shocks of different sizes (1, 5, 10, 15, 25%) to study the effects of the shock size on the IRF. Figure 5 and 6 represent the evolution of the IRF for the restricted STAR model of the entire sample and the IRF for the post Bretton-Woods sample model. The stability of the models is clear. The speed of convergence depends on the size of the shock, showing thus the non-linearity of the models. It is worth pointing out that the complete convergence of the models needs about 40 periods (i.e. 10 years) which confirms that the purchasing power parity is a long horizons phenomenon.

Figure 5: Impulse-response function (1960–1998)**Figure 6: Impulse-response function (1973–1998)**

4. CONCLUSIONS

The extensive literature on purchasing power parity has so far failed to give empirical support to the hypothesis when transaction costs are omitted. We test the adequacy of the nonlinear solution provided by the Dumas' model allowing for transaction costs for the Swiss case.

Applying the smooth transition regression method to the Swiss franc/German mark exchange rate, over the 1960–1998 period and the subperiod of the recent floating experience, we get stable ESTAR models. For small deviations from parity, the real exchange rate is described by a random walk and the deviations last for a long time. However, as the deviations become larger, the real exchange rate shows clear mean-reverting behaviour and can be represented by an $AR(5)$ model. The adjustment towards this steady state takes place very quickly. The stability analysis by a Monte Carlo simulation shows the clear mean-reverting and nonlinear properties of our models. The sub-period analysis shows that the nominal exchange rate regime does not affect the estimations, confirming the results of A.M. TAYLOR which shows that over the last century the capacity of the international market integration to smooth out real exchange rate shocks has not seriously changed over several exchange rate regimes.

The PPP deviations half-life are about 1.5 and 2.5 years for the restricted sample when real exchange rate are outside the no-arbitrage zone. These figures, are appreciably lower than values reported by most studies in this topic. As suggested by TAYLOR, this might be due to the elimination of the linearity bias.

An unambiguously overvaluation of the Swiss franc is detected during the 1970s oil crises and the Gulf War period. This result confirms the common belief that the Swiss franc is perceived as a safe instrument.

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SUMMARY

We test the hypothesis of nonlinear adjustment towards the purchasing power parity as suggested by Dumas' (1992) model. We estimate a stable exponential smooth transition regression model (ESTAR) for the Swiss franc/German mark exchange rate over the 1960–1998 period, where the adjustment to the steady state takes place rapidly. The results reveal that, for small deviations, the real exchange rate is best described by a random walk, whereas for larger deviations the real exchange rate is clearly mean-reverting. The same results are found when the sample is reduced to cover only the post Bretton-Woods period. A Monte Carlo simulation shows that our nonlinear models are clearly stable. When the real exchange rate is outside the no-arbitrage band, the esti-

mated deviation half-lives are about 1.5 and 2.5 years for respectively the entire and the restricted sample.

ZUSAMMENFASSUNG

Wir testen die Hypothese der nicht linearen Anpassung gegenüber der Kaufkraftparität wie beim Dumas (1992) vorgeschlagenen Model. Wir estimieren einen stabilen exponential smooth transition autoregressive (ESTAR) Model für den Schweizer Franken/Deutsch Mark Wechselkurs über die 1960–1998 Periode, in welcher die Anpassung schnell zu einem Aequilibrium führt. Die Resultate zeigen, dass für kleine Deviationen der reale Wechselkurs am besten durch einen Random Walk beschrieben werden kann, während für grössere Deviationen eine klare Tendenz, in den Mittelwert zurückzukehren, dargestellt wird. Die gleichen Resultate werden gefunden, wenn das Muster nur zur Periode nach Bretton-Woods reduziert wird. Eine Monte Carlo Simulation bestätigt die Stabilität unseres estimierten Models. Wenn der reale Wechselkurs sich ausserhalb der ohne-Arbitrage Zone befindet, ist die halbe Lebensdauer von 1.5 Jahr für das ganze Muster, und von 2.5 Jahren für das reduzierte Muster.

RÉSUMÉ

Nous testons l'hypothèse selon laquelle l'ajustement vers la parité des pouvoirs d'achat se fait d'une façon non linéaire, comme suggéré par le modèle de Dumas (1992). L'estimation d'un modèle autorégressif à transition lissée exponentiel (ESTAR) pour le taux de change franc suisse/mark allemand sur la période 1960–1998 indique que l'ajustement vers la parité se fait très rapidement. Pour des petites déviations de l'équilibre, le taux de change réel peut être représenté par une marche aléatoire, alors que pour des déviations plus importantes il montre une claire tendance au retour à la moyenne. Des résultats semblables sont obtenus lorsque l'échantillon est restreint à l'après Bretton-Woods. Une simulation de Monte Carlo confirme la stabilité du modèle estimé. Lorsque le taux de change réel est en dehors de la bande de non-arbitrage, sa demi-durée de vie est respectivement de 1,5 et 2,5 années pour l'échantillon complet et l'échantillon restreint.