Structural Breaks and the Normality of Stock Returns*

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1. Introduction

Modeling financial distributions has been regarded as one of the most important research areas in the asset pricing literature. Many financial academics have consequently expressed their concerns over the question of which distribution best fits actual return distributions. To name a few, such scholars of note include Blattberg and Gonedes (1974), Kon (1984), Kim and Kon (1996), Blazenko (1996), Knight and Satchell (2000), and Aparacio and Estrada (2001). A brief review of the previous research on this topic shows that earlier research has placed more emphasis on finding unconditional distributions which can explain non-normal properties such as skewness and excess kurtosis, now referred as stylized facts about actual stock return distributions. In contrast, recent research has focused more on finding appropriate models that can explain time-variations of both expected returns and volatility. The ARCH-type conditional models such as GARCH, GJR, and EGARCH models belong to this latter direction of research.

* job market paper

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One of the models previously suggested by finance scholars as an alternative to the long-standing normal distribution is the mixture-of-normals model. It was known to be first proposed by KON (1984) in the literature of asset pricing. In his paper on U.S. firms, KON claims that the actual distribution of stock returns is composed of several normal distributions. It has been recognized that skewness and excess kurtosis observed in actual return data can be explained by this discrete mixture-of-normals model. More recently, KIM and KON (1996) suggested the sequential mixture-of-normals model, and attempted to explain time-variations of expected returns and volatility.

In reference to their mixture-of-normals models, this paper aims at proposing another form of the mixture-of-normals model as an alternative. Under the concept of the structural break, the present model attempts to locate sub-periods during which normal distributions with different parameters hold. Thus, the time-varying properties of the expected returns and volatility will be incorporated into this model and such properties as non-normality will be explained, using the return data of the Swiss stock index. In addition, the statistical power of the derived methodology will be confirmed by implementing simulations. To do this, random numbers will be generated from various schemes of the mixtures of normal distributions with different parameters, and their statistical robustness will be calculated. The statistical power of the JARQUE-BERA (1980, hereafter J-B) statistic will also be checked under the same simulated experiments.

Compared to the previous models of KON (1984) and KIM and KON (1996), the present model poses the following similarities. First, the usage of the terminology of the mixture-of-normals model is the same in all three studies. Second, the methodology in this paper is similar to that of KIM and KON (1996) in that both used the sequential procedure in detecting break points. In spite of these similarities, this study shows the following differences. First, this research is motivated as an attempt to apply GOLDFELD and QUANDT's (1972) two-period structural break analysis, while KIM and KON (1996) placed more emphasis on comparison of the explanatory power of their sequential mixture-of-normals model with those of Student-t and Poisson jump models. Second, with the mixture-of-normals model of this paper, the time-varying properties of the expected returns and volatilities can be explained, while those properties are not explicitly explained with the mixture-of-normals model of KON (1984). In this respect, by proposing a new mixture-of-normals model, this research provides theoretical ground for the use of time-varying mean-variance models in the investment theory.

The rest of the paper proceeds as follows: Section 2 defines the concept of structural breaks, and derives a specific form under the mixture-of-normals framework. Data and empirical results are presented in Section 3. Section 4 checks for statistical power through simulations, and Section 5 summarizes and concludes the paper.

2. Theoretical Derivation

2.1 Definition of Structural Breaks

Modifying the definition of structural breaks given by Kim and Kon (1996), this study defines the concept as follows: When a sequence of observations $\{X_i\}$ exists and abrupt changes occur at points t_{r1}, t_{r2}, t_{r3}, …, $t_{r(r-1)}$, it is said that structural breaks exist at these points.

$$
\begin{array}{l}
X_i \cdots\cdots f(\theta_{r1}) \; i = 1, 2, \ldots, t_{r1} \\
X_i \cdots\cdots f(\theta_{r2}) \; i = t_{r1} + 1, t_{r1} + 2, \ldots, t_{r2} \\
\qquad\qquad\quad \ldots \\
\qquad\qquad\quad \ldots \\
X_i \cdots\cdots f(\theta_{rr}) \; i = t_{r(r-1)} + 1, t_{r(r-1)} + 2, \ldots, t_{rr} = T
\end{array}
$$

where θ_{ri} stands for parameters of the i-th sub-sequence.

2.2 Economic Significance of the Model

In this section, the economic significance of the preceding concept of the structural break will be discussed under the framework of the mixture of normal distributions, with the possibility of being applied to a return generating process.

First, the mixture-of-normals model can explain other non-normal distributions when its parameters are appropriately chosen. For example, McLachlan and Peel (2000) point out that a mixture model is able to represent quite complex distributions through an appropriate choice of its components to depict accurately the local areas of support of the true distribution. McLachlan and Peel (2000) further state that the mixture model can handle situations where a single parametric family is unable to provide a satisfactory model for local variations in the observed data. In this context, Priebe (1994) supports this position by showing that the log-normal distributions can be approximated using the mixtures of normal distributions. Second, these types of the mixture-of-normals models can explain skewness and excess kurtosis observed in actual data of stock returns. For instance, Assoe (1998) points out that the mixture-of-normals model is leptokurtic because a high-variance distribution expands the tails of low-variance distributions. And he also states that the mixture-of-normals model is skewed because the means differ across the distributions. Lastly, the present mixture-of-normals model can be viewed as a data generating process, a process which indicates that economic or political events might have occurred at the estimated structural break points, with significant impact on the return generating processes. That is, under the normality framework, the return generating process $N(\mu_{r1}, \sigma^2_{r1})$ begins from time 1 to time t_{r1}, and an event of impact occurs around the estimated break point t_{r1}, thus changing the ongoing return generating process into $N(\mu_{r2}, \sigma^2_{r2})$. This new process persists up to time t_{r2}, at which point the new process $N(\mu_{r3}, \sigma^2_{r3})$ emerges due to news around the estimated break point. In general, this method of logic can continue throughout the entire process. If the genuine return generating processes follow the mixture of normal distributions this way, the estimated break points can be theoretically linked with actual events in an institutional context, which are presumed to impact the return process.

2.3 Logic for the Estimation of Structural Break Points

The following procedures are taken in detecting structural break points as in QUANDT (1958, 1960). In the first stage, it is assumed that the time period consists of two sub-periods rather than one. Thus, it should be determined whether it is possible to explain the entire data set with only one distribution (e.g., no structural break) or whether it is statistically advantageous to explain the data with multiple distributions. In this stage, if the null hypothesis of no structural break is not rejected, the entire data set can be safely explained with only one distribution. However, if the null hypothesis is rejected, then there is the possibility that two or more sub-periods exist over the time period. In this second stage, a new null hypothesis that two sub-periods exist is set against the alternative hypothesis that three sub-periods exist. If the null hypothesis is not rejected, the search process ends at this stage. If the null hypothesis is rejected, however, the search process continues in the same manner. The number of sub-periods can be determined by repeating this search process.

2.4 Structural Breaks of the Normal Distribution

The estimation of structural break points of normal distributions will be conducted by the following maximum likelihood estimation method. The logic of QUANDT (1958, 1960), and GOLDFELD and QUANDT (1972) will be again used throughout this paper when estimating break points. Compared with the standard CHOW-type structural change test, the methodology of the preceding and present studies can be more econometrically appealing because heteroscedasticities in variances are allowed in the models. Notations are briefly defined as below:

μ_{rm}: the average of the variables during the *m*-th sub-period when the entire period is divided into *r* sub-periods ($r \geq m$),
σ_{rm}: the standard deviation of the variables during the *m*-th sub-period when the entire period is divided into r sub-periods,
t_{rm}: the possible break point of the *m*-th sub-period when the entire period is divided into r sub-periods, and
t^*_{rm}: the optimal estimated break point of the *m*-th sub-period when the entire period is divided into *r* sub-periods.

There are several steps in determining whether the entire data set can be explained by a single normal distribution or by two or more normal distributions. I first assume the former, which is equivalent to saying that no structural shift exists over the entire period. This null hypothesis that no structural change exists can be conceptualized by a random variable as stated below, and the likelihood function will be given by as in (1).

$$X_i \cdots\cdots\cdots N(\mu_{11}, \sigma^2_{11}) \; i = 1,2,\ldots, T$$

$$l_1 = \left(\frac{1}{(2\pi\sigma^2_{11})^{1/2}}\right)^T \exp\left(\frac{\sum_{i=1}^{T}(X_i - \mu_{11})^2}{2\sigma^2_{11}}\right) \tag{1}$$

Differentiating the logarithm of (1) with respect to $\sigma_{11}{}^2$, and substituting the result into (1) obtains the simple form of the likelihood function as in (2).

$$l_1 = \left(\frac{1}{(2\pi\sigma^2_{11})^{1/2}}\right)^T \exp\left(-\frac{T}{2}\right) \tag{2}$$

In contrast, when the structural break occurs during the period, two normal distributions with statistically different means (μ_{21}, μ_{22}) and variances (σ^2_{21}, σ^2_{22}) may exist during the two sub-periods.

$$X_i \cdots\cdots\cdots N(\mu_{21}, \sigma^2_{21}) \; \mathrm{i} = 1,2,\ldots, t_{21}$$
$$X_i \cdots\cdots\cdots N(\mu_{22}, \sigma^2_{22}) \; \mathrm{i} = t_{21} + 1, t_{21} + 2,\ldots, T$$

Now the null hypothesis that only one normal distribution holds is set against the alternative in which two normal distributions exist. Differentiating the log-likelihood function with respect to the variance parameters (σ^2_{21}, σ^2_{22}) and substituting them into the new likelihood function, combined with the likelihood function (2), leads to the likelihood ratio statistic of (3).

$$\lambda_{12} = \frac{l_1}{l_2} = \frac{\sigma_{21}^{t_{21}} \sigma_{22}^{T-t_{21}}}{\sigma_{11}^{T}} \tag{3}$$

The optimal break point t^*_{21} in the first stage is determined from the values of $-2\log\lambda_{12}$, which change over time. The largest value exceeding the critical value is regarded as the optimal break point t^*_{21} as in QUANDT (1958, 1960). The degree of freedom for the likelihood ratio (3) is three because the degrees of freedom of the numerator and denominator are $(T-5)$ and $(T-2)$, respectively. If the null hypothesis is rejected at this stage, the search process continues.

In the second stage, I investigate the possibility that more than two normal distributions with significantly different means and variances exist. I set the null hypothesis (H_0) for the existence of two normal distributions ($N(\mu_{21}, \sigma^2_{21})$, $N(\mu_{22}, \sigma^2_{22})$) against the alternative (H_1) for the existence of three normal distributions ($N(\mu_{31}, \sigma^2_{31})$, $N(\mu_{32}, \sigma^2_{32})$, $N(\mu_{33}, \sigma^2_{33})$). In this stage, by the same logic as mentioned earlier, the test statistic $-2\log\lambda_{23}$ will asymptotically follow a chi-square distribution with three degrees of freedom, with break points at the largest values of $-2\log\lambda_{23}$, which are considered optimal change points.

$$\begin{aligned}
H_0: \quad & X_i \cdots\cdots N(\mu_{21}, \sigma^2_{21}) \; i = 1, 2, \ldots, t_{21}^{*} \\
& X_i \cdots\cdots N(\mu_{22}, \sigma^2_{22}) \; i = t_{21}^{*} + 1, t_{21}^{*} + 2, \ldots, T \\
H_1: \quad & X_i \cdots\cdots N(\mu_{31}, \sigma^2_{31}) \; i = 1, 2, \ldots, \mathrm{t}_{31} \\
& X_i \cdots\cdots N(\mu_{32}, \sigma^2_{32}) \; i = t_{31} + 1, t_{31} + 2, \ldots, t_{32} \\
& X_i \cdots\cdots N(\mu_{33}, \sigma^2_{33}) \; i = t_{32} + 1, t_{32} + 2, \ldots, T
\end{aligned} \tag{4}$$

In this stage, following the same logic yields the simple form of the likelihood ratio of (5).

$$\lambda_{23} = \frac{l_2}{l_3} = \frac{\sigma_{31}^{t_{31}} \sigma_{32}^{t_{32}-t_{31}} \sigma_{33}^{T-t_{32}}}{\sigma_{21}^{t_{21}^{*}} \sigma_{22}^{T-t_{21}^{*}}} \tag{5}$$

If the null hypothesis is rejected, I test for the mixture of three normal distributions against the mixture of four normals hypothesis in the subsequent stage. The alternative hypothesis will be as follows.

$$
\begin{aligned}
H_1: \quad & X_i \cdots\cdots N(\mu_{41}, \sigma^2{}_{41}) \; i = 1, 2, \ldots, t_{41} \\
& X_i \cdots\cdots N(\mu_{42}, \sigma^2{}_{42}) \; i = t_{41} + 1, t_{41} + 2, \ldots, t_{42} \\
& X_i \cdots\cdots N(\mu_{43}, \sigma^2{}_{43}) \; i = t_{42} + 1, t_{42} + 2, \ldots, t_{43} \\
& X_i \cdots\cdots N(\mu_{44}, \sigma^2{}_{44}) \; i = t_{43} + 1, t_{43} + 2, \ldots, T
\end{aligned}
$$

Differentiating the log-likelihood function with regard to the variance parameters $(\sigma^2{}_{41,} \sigma^2{}_{42}, \sigma^2{}_{43,} \sigma^2{}_{44})$ and substituting them into a new likelihood function, combined with the likelihood function in the previous stage, leads to the likelihood ratio statistic of (6).

$$\lambda_{34} = \frac{l_3}{l_4} = \frac{\sigma_{41}^{t_{41}} \sigma_{42}^{t_{42}-t_{41}} \sigma_{43}^{t_{43}-t_{42}} \sigma_{44}^{T-t_{43}}}{\sigma_{31}^{t_{31}^*} \sigma_{32}^{t_{32}^*-t_{31}^*} \sigma_{33}^{T-t_{32}^*}} \tag{6}$$

Also in this stage, the test statistic -2logλ_{34} is asymptotically distributed as a χ^2 with 3 degrees of freedom. The change points with the largest value of -2logλ_{34} in excess of the critical value are regarded as the optimal change points t_{41}^* , t_{42}^* , and t_{43}^* .

Continuing this way, we can generalize this rationale. When a structural break happens $(r-1)$ times, there exists r sub-periods during which normal distributions with different means and variances hold. This concept of generalized structural break analysis for normal distributions can be expressed as in (7) and (8):

$$
\begin{aligned}
& X_i \cdots\cdots N(\mu_{r1}, \sigma^2{}_{r1}) \; i = 1, 2, \ldots, t_{r1} \\
& X_i \cdots\cdots N(\mu_{r2}, \sigma^2{}_{r2}) \; i = t_{r1} + 1, t_{r1} + 2, \ldots, t_{r2} \\
& \qquad\qquad \ldots \\
& X_i \cdots\cdots N(\mu_{rr}, \sigma^2{}_{rr}) \; i = t_{r(r-1)} + 1, t_{r(r-1)} + 2, \ldots, T
\end{aligned} \tag{7}
$$

$$\lambda_{(r-1)r} = \frac{\prod_{i=1}^{i=r} \sigma_{r1}^{t_{ri} - t_{r(r-1)}}}{\prod_{i=1}^{i=r-1} \sigma_{(r-1)r}^{t_{(r-1)i}^* - t^*_{(r-1)(i-1)}}} \tag{8}$$

where $t_{rr} = t^*_{(r-1)(r-1)} = T$, and $t_{r0} = t^*_{(r-1)0} = 0$.

The likelihood ratio in this generalized stage has three degrees of freedom, and the test statistic -2log$\lambda_{(r-1)r}$ follows a $\chi^2(3)$ distribution.

3. Empirical Results

3.1 Data

The source of the database used in this study is *Primark Datastream*. The monthly stock price index of Switzerland from July 1988 to December 2000 was used in this paper. There was a total of 150 return data points. Statistical tests for both simple and log-returns are implemented for the following reasons. First, portfolio theory calls for simple returns. Second, on a theoretical basis, log-returns are normally distributed, although they do not have a portfolio property.

The Swiss Market Index, from June 30, 1988, is a narrow-based index of 24 stock issues representing 20 highly capitalized companies listed on the Zurich, Geneva, and Basel stock exchanges (Berlin, 1990). Its base of 1,500 was assigned on June 30, 1988. As of October 2003, the number of companies in the SMI index was 26. Monthly data were used in carrying out empirical analyses because using daily or weekly data requires a tremendous number of computations. Without loss of generality, the monthly data were used in this research because the same logic can be applied to daily or weekly data.

3.2 Empirical Results

Table 1 reports the descriptive statistics of the variables and the J-B statistics for the index return data of the SWX Swiss Stock Exchange. I find that the monthly log and simple returns of the Swiss index show negative skewnesses of -0.784117 and -0.436655, respectively. Overall, the distribution of the returns on the Swiss index deviates from normality as the J-B statistics of 70.77587 and 44.53023 indicate.

Table 2 reports the application results of the derived methodology to the log returns of the Swiss index. The results indicate that the entire period of July 1988 to December 2000 is made up of four sub-periods. At the end of the first stage, the period in question can be divided into two sub-periods: July 1988 to July 1998 and August 1998 to December 2000. In the second stage, the divided sub-periods were further divided into three sub-periods: July 1988 to July 1999, August 1999 to May 2000, and June 2000 to December 2000. At the end of the third stage, the entire period was divided into four sub-periods (July 1988 to February 1991, March 1991 to May 1997, June 1997 to October 1998, and November 1998 to December 2000) during which statistically different normal distributions existed. In this stage, the test statistic reached a point 32.350 beyond the given critical value. Theoretically, this search process could continue, but

Table 1: Sample moments of the return distributions of monthly stock index

Statistics \ Variable	Log returns	Simple returns
Mean	0.012238	0.012731
Median	0.017306	0.017456
Maximum	0.191274	0.210791
Minimum	-0.211973	-0.191014
Std. Dev.	0.053685	0.053387
Skewness	-0.784117	-0.436655
Kurtosis	5.977378	5.522333
Jarque-Bera	70.77587	44.53023
(p-value)	(0.00000)	(0.00000)

Notes: This table shows the mean, median, maximum, minimum, standard deviation, skewness, and kurtosis of the monthly log and simple returns of the Swiss stock index, respectively. Skewness = m_3/s^3, and kurtosis = m_4/s^4-3, where m_i and s are the i-th central sample moment and the sample standard deviation of each distribution respectively. The Jarque-Bera test statistic is asymptotically distributed as a chi-square with two degrees of freedom under the null hypothesis of normality, and its critical value at the 5% and 1% significance levels are 5.99, and 9.21 respectively. The J-B statistic is given by the following expression:

$$Jarque-Bera = N\left[\frac{skewness^2}{6}+\frac{(kurtosis-3)^2}{24}\right]$$

where N is the number of observations in the sample.

the process is stopped here because of the tremendous number of computations required.

I interpret these results based on the size of the J-B statistic because the focus of this study is to confirm whether the normality assumption holds for each divided sub-period using the J-B statistic. The entire distribution greatly deviates from normality as the statistic 77.77587 indicates. As the computation process continues, however, there are different possibilities in the normality of the distributions in the sub-periods. The J-B statistics can be used to explain this phenomenon. The J-B statistics for the four sub-periods are, respectively, 0.3812, 1.1593, 2.2601, and 1.8374, thus indicating approximate normality of the distributions in the four sub-periods.

Table 2: Detecting structural break points (log or continuous returns)

Divided Sub-periods	Test Statistics
1st stage July 1988 – July 1998, August 1998 – December 2000 Jarque-Bera Statistics and (p-values): 4.2809 (0.1176), 11.5974 (0.0030)	$-2\log\lambda_{12} = 10.650^{a}$
2nd stage July 1988 – July 1999, August 1999 – May 2000, June 2000 – December 2000 Jarque-Bera Statistics and (p-values): 57.8421 (0.0000), 0.4408 (0.8022), 2.3166 (0.3140)	$-2\log\lambda_{23} = 925.62^{b}$
3rd stage July 1988 – February 1991, March 1991 – May 1997, June 1997 – October 1998, November 1998 – December 2000 Jarque-Bera Statistics and (p-values): 0.3812(0.8265), 1.1593 (0.5599), 2.2601 (0.3230), 1.8374 (0.3990)	$-2\log\lambda_{34} = 32.250^{b}$

Notes: This table shows the application results of the derived methodology to the log returns of the Swiss index. The results show that the entire period of July 1988 to December 2000 consists of four sub-periods: July 1988 – February 1991, March 1991 – May 1997, June 1997 – October 1998, and November 1998 – December 2000. The test statistic $-2\log\lambda_{ij}$ is asymptotically distributed as a chi-square with three degrees of freedom, and its critical values at the 5% and 1% significance levels are 7.81 and 11.34, respectively.

a Significant at the 5% level.
b Significant at the 1% level.

Table 3 shows the divided results when the simple returns were used in the same analysis. When compared with the results in Table 2, I find the following differences. First, at the end of the second stage, I find that the entire period of [July 1988 to December 2000] is made up of three sub-periods: [(July 1988 to July 1999), (August 1999 to December 1999), and (January 2000 to December 2000)]. The dividing process was stopped in the third stage because no significant structural break points were found. Second, the divided results at the end of the first stage were the same as those in Table 2 when the log-returns were used. However, there existed a difference in the value of the test statistic $-2\log\lambda_{12}$.

In Table 4, estimated break points are related to the developments of the Swiss stock market. The purpose of this attempt is to enhance economic significance

Table 3: Detecting structural break points (simple or discrete returns)

Divided Sub-periods	Test statistics
1st stage July 1988 – July 1998, August 1998 – December 2000 Jarque-Bera Statistics and (p-values): 4.2809 (0.1176), 11.5974 (0.0030)	$-2\log\lambda_{12} = 133.86^{b}$
2nd stage July 1988 – July 1999, August 1999 – December 1999, January 2000 – December 2000 Jarque-Bera Statistics and (p-values): 57.8421 (0.0000), 0.5421 (0.7627), 0.3005 (0.8605)	$-2\log\lambda_{23} = 936.09^{b}$
3rd stage No significant structural break points were found.	$-2\log\lambda_{34} <$ given critical values

Notes: This table shows the application results of the derived methodology to the simple returns of the Swiss index. The results show that the entire period of July 1988 to December 2000 consists of three sub-periods: July 1988 – July 1999, August 1999 – December 1999, and January 2000 – December 2000. The test statistic $-2\log\lambda_{ij}$ is asymptotically distributed as a chi-square with three degrees of freedom, and its critical values at the 5% and 1% significance levels are 7.81 and 11.34, respectively.

a Significant at the 5% level.
b Significant at the 1% level.

of the derived model by incorporating possible institutional effects on the return generating processes of the Swiss stock index. For example, we find that the rise of world's major equity indices might have influenced the return generating process of the Swiss market around October 1998. And we also find that, around January 2000, introduction of derivatives with the U.S. dollar or euro as trading currency might have affected the return process of the Swiss market. The estimated break points are also linked to the developments of the Swiss economy level. And the results are presented in Table 5, where we note that the formal confirmation of the independence of the SNB (Swiss National Bank) might have affected the return generating process around May/June 1997.

Table 4: Relating estimated break points to possible economic factors at the level of the SWX Swiss stock market

Estimated break points using		Possible impact factors at the SWX Swiss stock market level
log returns	simple returns	
February 1991 – March 1991		– At this time, the SWX Swiss Stock Exchange was not yet established.
May 1997 – June 1997		– IPO by Selecta R. – Banca del Gottardo is admitted as a new member of SWX. – IPO by Interroll Holding R. – IPO by Komax R. – UBS, Basle and Geneva, ceases trading at SOFFEX. – IPO by GretagMacbeth R.
October 1998 – November 1998		– After hitting new absolute lows for the year, the world's major equity market indices firm up again. – IPO of Swisscom. – Introduction of a closing auction for all SWX-listed shares. – The Board of SWX authorizes foreign institutions to commence trading.
	July 1999 – August 1999	– IPO of SC Turnaround Invest AG. – IPO of Agefi Groupe SA. – IPO of BioMarin Pharmaceutical Inc. – The Board of directors adopts a leaner organizational structure for the SWX Swiss Exchange.
	December 1999 – January 2000	– Pilot participants of the SWX Swiss Exchange and Paris Bourse trade in French$^{\text{SBF}}$ and Swiss equities and bonds for the first time (cross membership). – Introduction of derivatives with the US dollar or euro as the trading currency.

Notes: These events of possible impact are collected from the several issues of the *Annual Reports of the SWX Swiss Stock Exchange*.

Table 5: Relating estimated break points to possible economic factors at the level of the Swiss economy

Estimated break points using		Possible impact factors at the Swiss economy level
log returns	simple returns	
February 1991 – March 1991		– The maximum share of real estate in pension funds' portfolios is raised from 30 to 50 per cent. – The second Chamber of Parliament endorses the 10th revision of the old-age and disability insurance system. – The Parliament decides to support residential construction for lower income groups by an extra SF 1.7 billion between 1992 and 1996.
May 1997 – June 1997		– The Federal Council proposes to raise the standard VAT rate. – The Federal Council proposes a constitutional amendment which formally confirms the independence of the Swiss National Bank (SNB) and which defines the maintenance of price stability as SNB's primary objective.
October 1998 – November 1998		– The federal government proposal to reform the labor law is approved by referendum. – Swiss voters approve in a referendum the construction of two new transalpine rail tunnels.
	July 1999 – August 1999	– The Federal Council proposes a draft Federal Act on the Electricity market (June, 1999).
	December 1999 – January 2000	– The National Bank abandons its long-established practice of base money targeting and announces a new policy framework. – The Federal Parliament approves the budget of the Confederation for 2000.

Notes: These events of possible impact are collected from the several issues of the *OECD Economic Surveys, Switzerland*.

4. Test Power Checks

4.1 Power Checks for Structural Breaks

The statistical power for the derived methodology will be checked in two ways. The first way is to check the test power of the derived methodology by generating random distributions, repeating the procedure, and comparing the test statistics with the given critical value. The second way is to check the robustness of the J-B statistic in terms of the mixture-of-normals hypothesis using the same procedures as before.

Simulations are conducted and, more specifically, random numbers are generated from the mixture-of-normals distributions with varying parameters in order to check the statistical power of the derived methodology. In this process, the number of mixtures is restricted to fewer than five, that is, 2, 3, and 4. The reason for this is because increasing the number of mixtures exponentially increases the number of computations, so at a certain point, the number of combinations goes beyond the computing ability of present-day PCs. This process then checks how accurately the proposed methodology can detect underlying break points by generating 150 random numbers each time from the 100,000 simulated distributions. The number of data points at each point is fixed at 150 to enhance comparability with the preceding empirical tests. In these experiments, the test statistic $-2\log\lambda_{ij}$ is calculated and compared with the given critical value, $\chi^2(3)_{0.05}$ or $\chi^2(3)_{0.01}$. Depending on the significance level, the number of times the test statistic exceeds the given critical value ($\chi^2 > \chi^2(3)_{0.05}$ or $\chi^2(3)_{0.01}$) is calculated. This logic is similar to the method that the marginal significance level is obtained under the bootstrapping method.

In carrying out experiments, the following procedures are performed each time:

(1) Generate 150 data points from normal distributions whose means and variances follow uniform (-1,1) and uniform (0,1) distributions, respectively.
(2) Generate 150 data points from normal distributions whose means and variances follow standard normal distributions.
(3) Generate 150 data points from normal distributions with zero means and different variances. These standard deviations of the normal distributions are first generated randomly from the standard normal distribution.
(4) Generate 150 data points from the normal distributions with zero means and different variances. These variances of the normal distributions are first generated randomly from the uniform (0,1) distribution.

Table 6: Simulation results of detecting change points

# of normals / statistics	$N=2$ (100,000)	$N=3$ (100,000)	$N=4$ (100,000)
(1) Average	42.79948 a	117.97601	144.68933
S.D.	12.92272 b	136.16384	274.73983
5%	100,000 c	76,774	67,662
1%	98,933 d	75,804	67,219
(2) Average	43.28274	201.38653	41.11793
S.D.	13.0540	273.20878	277.01352
5%	100,000	74,541	55,100
1%	99,076	74,157	54,636
(3) Average	45.32101	205.47514	69.87509
S.D.	13.86009	289.53604	291.03737
5%	100,000	73,459	58,147
1%	99,366	73,076	56,053
(4) Average	43.88232	158.55998	58.95878
S.D.	13.17109	257.96701	291.03737
5%	100,000	71,050	56,465
1%	99,176	70,580	56,053
(5) Average	42.08685	90.02183	262.74155
S.D.	12.80252	246.42345	303.84642
5%	100,000	64,801	78,422
1%	98,683	64,219	78,081

Notes: The table reports the average values of the test statistic $-2\log\lambda_{ij}$, their standard deviation (S.D.), and the number of times this test statistic exceeds the given significance levels. The experiment ($N=2$, scheme (1)) shows that the average value of $-2\log\lambda_{12}$ is 42.79948 as in (a), and its standard deviation is 12.92272 as in (b). The notation (c) indicates that in all 100,000 experiments, the value of $-2\log\lambda_{12}$ exceeds the given critical value at the 5% significance level. The notation (d) can be interpreted in the same way that in 98,933 of the 100,000 simulation experiments, the value of $-2\log\lambda_{12}$ exceeds the given critical value at the 1% significance level.

(5) Generate 150 data points from the normal distributions with varying means and the same variance. The variance of the normal distributions is 0.01^2, with means generated from the uniform (-1,1) distribution.

Table 6 reports the results of these schemes. The cell (1,1) can be used for explanation. The first experiment ($N=2$, scheme (1)) shows that the average value of $-2\log\lambda_{12}$ is 42.79948 as in (a), with a standard deviation of 12.92272 as in (b).

The notation (c) indicates that in all 100,000 experiments, the value of -2logλ_{12} exceeds the given critical value at the 5% significance level. The notation (d) can be interpreted in the same way: in 98,933 of the 100,000 simulations, the value of -2logλ_{12} exceeds the given critical value at the 1% significance level. Generally speaking, the statistical power of the developed method can be confirmed in this first set-up ($N = 2$). The test power reached 100% at the 5% significance level, and more than 98% at the 1% significance level. In the second set-up ($N = 3$), the test power ranged from 64% to 77%, and in the third set-up ($N = 4$), from approximately 55% to 78%, depending on the schemes.

4.2 Robustness Checks for Normality

The second approach is to confirm the normality of the randomly generated data in terms of the J-B statistic. The J-B statistic for the simulated data from the mixture of normal distributions is computed and the number of times the J-B statistic exceeds the given critical chi-square values is checked. If the average value of the J-B statistic exceeds the given critical level, the following possibility exists: The data for the entire period deviate from normality even though the period consists of more than one mixture of sub-periods in which different normal distributions exist. That is, the data for each sub-period are normally distributed, but the observed distribution of the data for the entire period is not normally distributed. If the statistical power of the procedure is confirmed, it can be safely stated that this structural break analysis-based approach, which divides the entire period into several sub-periods, is statistically supported.

The same procedure as before is used in carrying out the second experiment where 150 data points from the simulated distributions with varying parameters are generated each time, with the resultant J-B statistic computed. In this experiment, I then compare the statistic with the given critical level, $\chi^2(2)_{0.05}$ or $\chi^2(2)_{0.01}$, and compute the number of times the test statistic is in excess of the given critical values, that is, $\chi^2 > \chi^2(2)_{0.05}$ or $\chi^2(2)_{0.01}$. Two degrees of freedom are used here because the J-B statistic is asymptotically distributed as a chi-square with two degrees of freedom. This is the method in which the power of non-normality for the entire distribution is confirmed by the J-B statistic. The same database as in schemes (1) through (5) is used.

Table 7 presents the J-B simulation results. In each cell, we note the average value of the J-B statistics, their standard deviation, and the number of times the test statistic -2logλ_{ij} is in excess of the given critical values at the 5% and 1% significance levels. For example, the second experiment (scheme (2) and $N = 3$) shows

Table 7: Simulation results for Jarque-Bera statistics

# of normals / statistics	$N=2$ (100,000)	$N=3$ (100,000)	$N=4$ (100,000)
(1) Average	2.73648	2.7259	2.71229
S.D.	7.04527	7.49386	7.19166
5%	10,408	10,282	10,303
1%	6,435	6,321	6,340
(2) Average	698.18253	668.60130 e	740.13449
S.D.	9457.6140	4792.32315 f	14915.7798
5%	90,542	90,430 g	90,524
1%	86,995	86,920 h	87,007
(3) Average	281.22844	281.44185	272.15690
S.D.	1357.71392	2099.90292	1102.02438
5%	99,325	99,310	99,255
1%	98,433	98,365	98,315
(4) Average	36.26763	36.51653	36.62842
S.D.	51.38950	53.90088	53.39210
5%	86,624	86,898	86,966
1%	77,924	78,144	78,065
(5) Average	8.13030	8.13478	8.14578
S.D.	1.53622	1.54216	1.54177
5%	91,508	91,448	91,668
1%	24,347	24,699	24,994

Notes: Table 7 reports the average value of the J-B statistics, their standard deviation (S.D.), and the number of times the test statistic exceeds the given critical values. The experiment (scheme (2) and $N=3$) shows that the average value of 100,000 J-B statistics is 668.60130 as in (e) and that their standard deviation is 4792.32315 as in (f). Of the 100,000 experiments, the J-B statistic exceeded the given critical values 90,430 (g) and 86,920(h) times, at the 5% and 1% significance levels respectively.

that the average value for 100,000 J-B statistics is 668.60130 with a standard deviation 4792.32315. Of the 100,000 experiments, the J-B statistic exceeded the given critical values 90,430 and 86,920 times, respectively, at the 5% and 1% significance levels.

The preceding results provide evidence that, though the J-B statistic for the entire data set greatly exceeds the normality level, the actual distribution could be made up of two or more underlying normal distributions.

5. Summary and Conclusion

This paper first defined the concept of structural breaks, and derived a special form based on the mixture-of-normals hypothesis. This study then applied the developed methodology to Swiss stock index returns. The results showed that the entire period was made up of three or four sub-periods during which statistically different normal distributions existed. As inferred from the J-B statistics, each sub-period's distribution was normal, despite the fact that all of the Swiss market index data was non-normal. This finding was interpreted as evidence supporting the derived methodology.

The research continued to check the statistical robustness of the methodology in two ways. First, random numbers were generated from the various forms of the mixture of normal distributions, and the test power of this experiment was calculated. Favorable results supporting the methodology were obtained. Second, I observed the change in the J-B statistics by generating random numbers. The results indicated the possibility that the underlying distribution was made up of a mixture of normal distributions, although the J-B statistic for the entire data set was beyond the normal range. These results could be interpreted as possible evidence for the mixture-of-normals hypothesis, previously suggested by KON (1984) and subsequent research.

The relative position and the contributions of this study can be highlighted in comparison with the previous research on the stock return distribution. The position of this research is that the observed non-normality of the actual distribution could be the combined result of normal distributions with time-varying parameters. Thus, the major contribution of this study is that the present mixture-of-normals model can be used as an alternative explanation for the stylized facts of the actual stock return distributions: (1) non-normality, (2) time variation of expected returns, and (3) time variation of volatility. The observed excess kurtosis could be a result from the mixture of the distributions with high- and low variances. At the same time, the observed skewness could be another result from the mixture of the distributions with different means. The time variations of both expected returns and volatility could be explained by the time-varying parameters of the normal distributions.

Other minor contributions of the present study can be additionally stated as follows. First, the mixture-of-normals model in this study is the result from the application and extension of the GOLDFELD and QUANDT's (1972) two-period linear regression model. Second, the test power of the methodology has been also checked using simulations. Third, while emphasizing the statistical aspect, this study also contributes to the literature of portfolio investment by presenting another type of the mixture-of-normals model, thus supporting the use of time-varying M-V (Mean-Variance) models. Fourth, this research attempted to relate estimated breakpoints to the developments at the SWX Swiss Stock Exchange and of the Swiss economy in order to consider possible institutional impact on the return generating process.

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SUMMARY

This paper attempts to explain the distribution of actual stock index returns using a mixture of the normal distributions model. This paper first defines the concept of structural breaks and derives a special form of structural breaks under the normality framework. It then applies the derived methodology to the monthly returns of the Swiss stock index to confirm whether the observed non-normality of stock returns can be explained with the derived model. Empirical results provide evidence that the entire period consists of three or four sub-periods in which different normal distributions exist. To check the statistical power of the model, this study generates random data from the normal distributions. Simulation results support the statistical power of the new methodology, and indicate the possibility that, despite being a seemingly non-normal test statistic for the entire data set, the underlying distribution is made up of a mixture of normal distributions.

ZUSAMMENFASSUNG

In der vorliegenden Arbeit versucht der Verfasser, die Streuung des Ertragswertes von wirklichen Aktien mit Hilfe des Mixturmodells der mehreren normalen Streuungen zu erklären. In dieser Arbeit wird zunächst der Begriff von Strukturwandel definiert und eine spezifische Form des Strukturwandels unter dem Rahmen der Normalität abgeleitet. Anschließend wird die abgeleitete Methode auf den monatlichen Ertragswert des schweizerischen Aktienindex angewendet, um zu überprüfen, ob die anscheinend beobachtete Abnormalität mit Hilfe des in dieser Arbeit abgeleiteten Modells erklärt werden kann. In der empirischen Untersuchung wird es gezeigt, dass die Gesamtperoide aus drei oder vier Subperioden besteht, wo verschiedene Mittelwerte und Streuungen vorhanden sind. Der Verfasser hat einen Simulationstest gemacht, indem er Zufallzahlen erzeugt hat, um die statistische Zuverlässigkeit zu überprüfen. Der Simulationstest hat die statistische Zuverlässigkeit der neu abgeleiteten Methode bestätigt und außerdem gezeigt, dass die normale Streuung in der Wirklichkeit Ergebnis von mehreren Mixturmodellen sein kann, obwohl die Gesamtdaten eine abnormale Streuung zu zeigen scheinen.

RÉSUMÉ

Cet article essaye d'expliquer les distributions des retours d'actions courantes en se basant sur un mélange de modèles de distributions normales. Cet article définit d'abord le concept des coupures structurales d'où dérive une forme spéciale de coupures structurales dans un cadre normalisé. Cette étude applique alors la méthodologie dérivée pour les retours mensuels de l'indice des actions Suisse pour confirmer si la non-normalité observée des retours courants peut être expliquée avec le modèle dérivé. Les résultats empiriques fournissent la preuve que la période entière se décompose en trois ou quatre sous-périodes où les différentes distributions normales sont présentes. Pour vérifier la puissance statistique de ce modèle, cette étude génère des données aléatoires à partir du mélange des distributions normales. Les résultats de la simulation montrent l'existence de la puissance statistique de la nouvelle méthodologie dérivée, et indiquent la possibilité que, en dépit d'être un essai statistique apparemment non-normal pour l'ensemble complet des données, la véritable distribution prioritaire se compose d'un mélange de distributions normales.